A REDUCED-COMPLEXITY PTS SCHEME FOR PEAK-TO-AVERAGE POWER RATIO REDUCTION IN OFDM SYSTEMS

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ABSTRACT

In this paper a reduced-complexity partial transmit sequences (PTS) approach is proposed for the reduction of peak-to-average power ratio (PAPR) in OFDM systems. In the PTS scheme, we need several IFFTs (inverse fast Fourier transforms) to compute the partial transmit sequences. There are two main contributions in the paper. Firstly, we propose to partition the input symbols into disjoint subblocks in an interleaving manner. In this way, we can significantly reduce the complexity by sharing the arithmetic in the IFFT computations. Secondly, we show that in the new partition scheme, the number of rotation factors can be reduced by one without affecting the PAPR reduction ability. As a result, the search time for the optimal rotation factor can be reduced by 75%. Simulation results show that the PAPR of the proposed method is only slightly worse than the original PTS scheme but it has a much lower complexity.

1. INTRODUCTION

One major drawback of the orthogonal frequency division multiplexing (OFDM) system is its high peak-to-average power ratio (PAPR). In OFDM systems we use the inverse discrete Fourier transform (IDFT) to process the baseband signals. After applying the IDFT, the envelope of the transmitted samples is approximately Rayleigh-distributed, and the transmitted samples can have a high PAPR.

In the literature, many distortionless methods have been proposed for the PAPR reduction [1]-[7]. One effective approach for PAPR reduction is to generate a number of candidates of transmitted signals and select the one with the smallest PAPR for transmission. The selective mapping (SLM) method [1][2] and the partial transmit sequences (PTS) method [4]-[7] belong to this approach. In the SLM method, the candidates are generated by multiplying the input modulation symbols by different rotation factors which are often chosen from the set $\{\pm 1, \pm j\}$. However, the SLM method has a high complexity. To generate M candidates, one has to compute M NL-point IDFT, where N is the block size and Lis the oversampling factor. Though the IDFTs can be efficiently implemented using inverse fast Fourier transform (IFFT) when N and L are powers of 2, the computational complexity can still be too high for many applications, especially when M is large. To reduce the complexity, the PTS method divides the input modulation symbols into a few disjoint subblocks and each subblock is weighted by a rotation factor. By doing so, the PTS scheme needs only $(\log_4 M + 1) NL$ -point IFFTs (where M is number of candidates). In [5], a low complexity algorithm is proposed for the optimization of the rotation factors in the PTS approach. In [6], the authors proposed a gradient descent search method to find the rotation factors. In [7], an improved rotation factor computation is introduced. Though the methods in [5][6][7] can greatly lower the cost of rotation factor optimization, the number of IFFTs needed is not reduced.

In this paper, a new method is introduced to further reduce the complexity of the PTS scheme. By exploiting the decimation-intime (DIT) IFFT algorithm, we will first show that we can share most of the computation in the $(\log_4 M + 1) NL$ -point IFFTs. In the special case of four partial transmit sequences, the total number of multiplications needed for computing all four PTS is the same as that of one NL-point IFFT. Moreover we will show that for the proposed method, we can set the first two rotation factors (rather than one) without affecting the PAPR reduction ability of the PTS scheme. Thus the cost of the rotation factor optimization is reduced to a quarter of that in the original PTS scheme. Simulation results show that the PAPR reduction performance of the proposed PTS scheme is only slightly worse (less than 0.5dB) than the original PTS scheme but its computational complexity is much lower.

The paper is outlined as follows. Sec. 2 describes the SLM and the PTS approaches. In Sec. 3, the proposed PTS approach is presented. Simulation results are given in Sec. 4 and conclusions are given in Sec. 5.

Notation: Boldfaced lower case and upper case letters represent vectors and matrices respectively. The notation \mathbf{A}^{\dagger} denotes the transpose-conjugate of \mathbf{A} and \mathbf{A}^{T} denotes the transpose of \mathbf{A} . As the scaling factor does not change the PAPR, for notation simplicity, we drop the normalization factor in the IDFT. The $N \times N$ IDFT matrix \mathbf{Q}_{N} is defined as $[\mathbf{Q}_{N}]_{k,l} = e^{j\frac{2\pi}{N}kl}$. The Kronecker product of an $m \times n$ matrix \mathbf{A} and an $p \times q$ matrix \mathbf{B} is an $mp \times nq$ matrix given by:

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{0,0}\mathbf{B} & \cdots & a_{0,n-1}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{m-1,0}\mathbf{B} & \cdots & a_{m-1,n-1}\mathbf{B} \end{bmatrix}$$

Throughout the paper, the IDFT size N is assumed to be a power of 2 so that IFFT can be employed for its implementation. When a sequence has fewer than N samples, zeros are padded to the sequence for its N-point IDFT computation. For convenience, $N \times 1$ vectors and length-N sequences are used interchangeably.

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2. THE SLM SCHEME AND PTS SCHEME

In this section, we briefly review two techniques, namely the selective mapping (SLM) [1][2] and the partial transmit sequences (PTS) [4], for PAPR reduction in OFDM systems. Let N be the input block size of the OFDM system. That means, one OFDM input block consists of N modulation symbols s_k and N is also the size of inverse DFT at the transmitter. In this paper, we denote the OFDM input block by an $N \times 1$ vector $\mathbf{s} = \begin{bmatrix} s_0 & s_1 & \cdots & s_{N-1} \end{bmatrix}$ In an OFDM system, each symbol s_k is modulated by a different orthogonal subcarrier, and the signal sent at the transmitter is obtained by summing up these N modulated carriers:

$$x(t) = \sum_{n=0}^{N-1} s_n e^{j2\pi n \, \triangle ft}, \quad 0 \le t < NT, \tag{1}$$

where $\Delta f = \frac{1}{NT}$ is the frequency spacing of the subcarriers. In practice, most OFDM systems use a discrete-time implementation for baseband processing. For the computation of PAPR [7], we usually oversample x(t) by a factor of L to obtain the discrete-time signal

$$x_k = \sum_{n=0}^{N-1} s_n e^{j2\pi k n \, \Delta f T/L} \,, \quad k = 0, \, 1, \cdots, \, LN - 1. \tag{2}$$

The sequence x_k can be interpreted as the *NL*-point IDFT of the input vector **s** padded with (L - 1)N zeros. The PAPR of x_k or the vector $\mathbf{x} = [x_0 \dots x_{NL-1}]^T$ is given by

$$PAPR(\mathbf{x}) = \frac{\max_{0 \le k \le NL-1} |x_k|^2}{E[|x_k|^2]}.$$
 (3)

It is known [7] that when the oversampling factor L is ≥ 4 , the PAPR of x(t) can be accurately approximated by the PAPR of x_k . One commonly used performance measure for PAPR reduction is the complementary cumulative distribution function(CCDF) which is defined as:

$$CCDF(PAPR_0) = Prob(PAPR > PAPR_0).$$
⁽⁴⁾

It is also known as the clipping probability.

The SLM Method [1][2]: In the selective mapping (SLM) approach, M independent candidates of the sequence x_k are generated and the one with the lowest PAPR is transmitted. To generate these candidates, we multiply the modulation symbol s_n by randomly generated unit-magnitude constant b_n . These constants b_n are also known as rotation factors. For simplicity, we usually choose $b_n \in \{\pm 1, \pm j\}$. The candidate sequences x_k are obtained by applying IFFT to the product $b_n s_n$. One can immediately see that the SLM method has a very high complexity. In general a total of M NL-point IFFTs are needed to generate M independent candidate sequences. One way to reduce the complexity is to use the PTS method, to be described next.

The PTS Method [4]: In the PTS approach, we partition the OFDM input block s into U disjoint subblocks \mathbf{s}_u , for $0 \le u \le U - 1$. In other words, if the *i*-th entry of \mathbf{s}_j is nonzero, the *i*-th entry of \mathbf{s}_l is equal to zero for all $l \ne j$. Moreover we have $\mathbf{s} = \sum_{u=0}^{U-1} \mathbf{s}_u$. Applying the *NL*-point IFFT to \mathbf{s}_u (padded

with zeros), we obtain \mathbf{x}_u , which is known as the *partial trans*mit sequence. Each \mathbf{x}_u is multiplied by a rotation factor b_u where $b_u \in \{\pm 1, \pm j\}$. The candidate signal is given by

$$\mathbf{x} = \sum_{u=0}^{U-1} b_u \mathbf{x}_u, \qquad b_u \in \{\pm 1, \pm j\}.$$
 (5)

One can see that the vector **x** is in fact the *NL*-point IFFT of the $T \sup_{u=0}^{U-1} b_u \mathbf{s}_u$. In the PTS scheme, all the entries in \mathbf{s}_u are multiplied by the same rotation b_u . It is clear that the PTS method is a special case of the SLM method. For a PTS scheme with U partial transmit sequences, the number of candidate sequences **x** are 4^{U-1} (the first rotation factor b_0 can be set to 1 without affecting the PAPR). Compared with the SLM method, the number of IFFT required in the PTS method is greatly reduced. To get $M = 4^{U-1}$ candidate sequences, we need to carry out U (or equivalent $\log_4 M + 1$) *NL*-point IFFTs. After the partial transmit sequences as in (5). A direct computation of (5) needs $M \cdot NL \cdot (U-1)$ additions. Below we will show how to further reduce the complexity of IFFT computation by carefully partitioning the input block **s** into \mathbf{s}_u .

3. PROPOSED REDUCED-COMPLEXITY PTS METHOD

In this section, we will first show how we can further reduce the complexity of the PTS method by partitioning the input block s into *U* disjoint subblocks s_i in an interleaving manner. Then we will prove that when the input is partitioned in this way, we can set the first two rotation factors $b_0 = b_1 = 1$ without affecting the PAPR reduction ability. Because $b_i \in \{\pm 1, \pm j\}$ and we have one fewer free parameters, the complexity of searching for the optimal rotation factors is reduced to 1/4 of the original PTS scheme.

To explain the idea, we take U = 4 as an example. And for simplicity, we assume L = 1, that is, there is no oversampling. In this case, the 4 subblocks are given by

$$\mathbf{s}_{0} = [s_{0} \ 0 \ 0 \ s_{4} \ 0 \ 0 \ s_{8} \ \cdots]^{T}
\mathbf{s}_{1} = [0 \ s_{1} \ 0 \ 0 \ s_{5} \ 0 \ 0 \ s_{9} \ \cdots]^{T}
\mathbf{s}_{2} = [0 \ 0 \ s_{2} \ 0 \ 0 \ s_{6} \ 0 \ 0 \ s_{10} \ \cdots]^{T}
\mathbf{s}_{3} = [0 \ 0 \ s_{3} \ 0 \ 0 \ s_{7} \ 0 \ 0 \ s_{11} \ \cdots]^{T}.$$
(6)

The partial transmit sequences \mathbf{x}_u are obtained by taking N-point IFFT of \mathbf{s}_u and the candidate sequence \mathbf{x} is obtained by taking the sum

$$\mathbf{x} = \sum_{i=0}^{3} b_i \mathbf{x}_i = \sum_{i=0}^{3} b_i \mathbf{Q}_N \mathbf{s}_i,$$
(7)

where \mathbf{Q}_N is the $N \times N$ IDFT matrix. The above implementation needs 4 N-point IFFTs. Suppose now that the N-point IDFT is implemented using the radix-4 decimation-in-time(DIT) IFFT algorithm [8]. Then \mathbf{Q}_N can be decomposed as a cascade of $\mathbf{Q}_{\frac{N}{4}}$ and \mathbf{Q}_4 as

$$\mathbf{Q}_{N} = \left(\mathbf{Q}_{4} \otimes \mathbf{I}_{\frac{N}{4}}\right) \mathbf{D} \left(\mathbf{I}_{4} \otimes \mathbf{Q}_{\frac{N}{4}}\right), \tag{8}$$

where **D** is a diagonal matrix consisting of the twiddle factors:

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{D}_3 \end{bmatrix}, \text{ with } [\mathbf{D}_i]_{nn} = e^{j\frac{2\pi kni}{N}}.$$
 (9)

Using the above decomposition and the special forms of s_i in (6), we can write

$$\mathbf{x} = \sum_{i=0}^{3} \mathbf{Q}_{N} b_{i} \mathbf{s}_{i} = \left(\mathbf{Q}_{4} \otimes \mathbf{I}_{\frac{N}{4}}\right) \cdot \begin{bmatrix} b_{0} \mathbf{D}_{0} \mathbf{Q}_{\frac{N}{4}} \mathbf{s}'_{0} \\ b_{1} \mathbf{D}_{1} \mathbf{Q}_{\frac{N}{4}} \mathbf{s}'_{1} \\ b_{2} \mathbf{D}_{2} \mathbf{Q}_{\frac{N}{4}} \mathbf{s}'_{2} \\ b_{3} \mathbf{D}_{3} \mathbf{Q}_{\frac{N}{2}} \mathbf{s}'_{3} \end{bmatrix}, \quad (10)$$

where $\mathbf{s}'_i = \begin{bmatrix} s_i & s_{4+i} & s_{8+i} & \cdots \end{bmatrix}^T$ is the four-fold downsampled version of the subblock \mathbf{s}_i in (6). From the right hand side of (10), we see that to get \mathbf{x} we need to implement four $\frac{N}{4}$ point IDFTs rather than four *N*-point IDFTs as in the original PTS scheme. Moreover when b_i changes, we do not need to recompute the vectors $\mathbf{D}_i \mathbf{Q}_{\underline{N}} \mathbf{s}'_i$. As \mathbf{Q}_4 has the form

$$\mathbf{Q}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

only additions are needed to compute \mathbf{x} when b_i changes. For the case of the oversampling factor L > 1, the same is true. The low complexity implementation of the proposed PTS scheme using the radix-4 NL-point IFFT is given in Fig. 1 for the case of U = 4. Notice that we need to implement only four $\frac{NL}{4}$ -point IFFTs, plus some twiddle factor α^i and the 4 \times 4 butterflies which need no multiplications. When the rotation factors b_i change, we do not need to recompute the vector \mathbf{v}_i (see Fig. 1) and only additions are needed to compute the candidate sequence \mathbf{x} from \mathbf{v}_i . The total number of multiplications in four NL/4-point IFFTs are the same as one NL-point IFFTs. Thus the complexity is reduced from four NL-point IFFTs in the original PTS scheme to one NL-point IFFT. The same technique can be applied to the more general case of U subblocks when U is a power of 2. By using the DIT IFFT structure, we can split an NL-point IFFT into a cascade of NL/Upoint IFFTs and U-point IFFTs. Like the case of U = 4, when b_i changes, there is no need to recompute the NL/U-point IFFTs and we need only to recompute the U-point IFFTs. In practice, Uis usually small, so the complexity of the U-point IFFTs is small. The complexity is thus reduced significantly.

Next we consider the rotation factors. In the original PTS scheme, the first rotation factor b_0 can be set to 1 without affecting the PAPR reduction ability. Below we will show that if the disjoint subblocks s_i are obtained by partitioning s in an interleaving manner, then we can set $b_0 = b_1 = 1$ without affecting the PAPR reduction ability. This result is proven in the next two theorems.

Theorem 1. For any integer n and any $U = 2^m \ge 4$, we have

$$\mathbf{Q}_U \mathbf{\Gamma}^n = \mathbf{P} \mathbf{Q}_U, \tag{11}$$

where $\Gamma = diag[1 \ j \ j^2 \ \cdots \ j^{(U-1)}]$ and **P** is a permutation matrix.

Proof: First let us define the diagonal matrix

$$\mathbf{\Lambda} = \operatorname{diag}[1 \ e^{j2\pi/U} \ e^{2\times j2\pi/U} \ \cdots \ e^{(U-1)\times j2\pi/U}].$$

The one can verify by direct multiplication that:

$$\mathbf{Q}_U \mathbf{\Lambda} = \mathbf{P}_1 \mathbf{Q}_U, \tag{12}$$

where $\mathbf{P}_1 = \begin{bmatrix} \mathbf{0}_{(U-1)\times 1} & \mathbf{I}_{(U-1)\times (U-1)} \\ 1 & \mathbf{0}_{1\times (U-1)} \end{bmatrix}$. Because $U = 2^m \ge 4$, U/4 will also be an integer. From (12) we have

$$\mathbf{Q}_U \mathbf{\Lambda}^{nU/4} = \mathbf{P}_1^{nU/4} \mathbf{Q}_U.$$

As $\mathbf{\Lambda}^{U/4} = \mathbf{\Gamma}$ and $\mathbf{P}_1^{nU/4}$ is a permutation matrix, we have proved (11).

Theorem 2. For any set of rotation factors $\{1, b_1, b_2, \dots, b_{U-1}\}$, where $b_i \in \{\pm 1, \pm j\}$, the corresponding candidate sequence **x** has the same PAPR as the candidate sequence **x'** generated by using another set of rotation factors $\{1, 1, b'_2, \dots, b'_{U-1}\}$ for some $b'_i \in \{\pm 1, \pm j\}$.

Proof: To prove the theorem, we first explain that it is sufficient to prove that for any set of rotation factors $\{1, b_1, \dots, b_{U-1}\}$,

$$\mathbf{Q}_{U} \operatorname{diag}[1 \ b_{1} \ b_{2} \ \cdots \ b_{U-1}] = \mathbf{P} \mathbf{Q}_{U} \operatorname{diag}[1 \ 1 \ b_{2}' \ \cdots \ b_{U-1}'], \ (13)$$

for some permutation matrix **P** and some b'_i . To see this, we look at Fig. 1 where the case of U = 4 is shown. From the figure, we see that the candidate sequence **x** generated by $\{1, b_1, \dots, b_{U-1}\}$ can be obtained by multiplying the entries of **v**_i by the matrix on the left hand side of (13). It is clear that if (13) holds, then the candidate sequence **x** generated by any $\{1, b_1, b_2, \dots, b_{U-1}\}$ is simply a permuted version of the candidate sequence **x'** generated by using $\{1, 1, b'_2, \dots, b'_{U-1}\}$. As a permutation does not change the PAPR of a sequence, the sequences **x** and **x'** have the same PAPR.

Next we will show (13). For any $\{1, b_1, b_2, \dots, b_{U-1}\}$, because $b_i \in \{\pm 1, \pm j\}$ we can always write

diag
$$[1 \ b_1 \ b_2 \ \cdots \ b_{U-1}] = \mathbf{\Gamma}^n$$
diag $[1 \ 1 \ b'_2 \ \cdots \ b'_{U-1}],$ (14)

for some *n* and some $b'_i \in \{\pm 1, \pm j\}$. Multiplying both sides of (14) by the IDFT matrix \mathbf{Q}_U , we have

$$\mathbf{Q}_U \operatorname{diag}[1 \ b_1 \ b_2 \ \cdots \ b_{U-1}] = \mathbf{Q}_U \mathbf{\Gamma}^n \operatorname{diag}[1 \ 1 \ b'_2 \ \cdots \ b'_{U-1}], \ (15)$$

Using the result of Theorem 1, (13) follows immediately from the above expression.

As b'_0 and b'_1 can be set to 1, we need to search for the optimal b'_2, \ldots, b'_{U-1} . The cost of rotation factor optimization is reduced to a quarter of the original PTS scheme.

4. COMPUTER SIMULATION

We carry out Monte-Carlo experiments to verify the performance of the proposed method. We consider two cases:

- 1. The block size is N = 64 and the modulation symbols are 16-QAM.
- 2. The block size is N = 256 and the modulation symbols are QPSK.

The oversampling factor is L = 4. A total of 100000 random OFDM blocks are generated. We will compare our results with the SLM method with M = 16 and the PTS method with $\{U = 4, M = 64\}$. The rotation factors of all methods are optimally chosen from the set $\{\pm 1, \pm j\}$ and they are obtained by using an exhaustive search. For comparison, we also include the PTS approach in [5] with $\{U = 4, M = 8\}$ and $\{U = 8, M = 16\}$. For our proposed PTS method, we consider the two cases of $\{U = 4, M = 16\}$ and $\{U = 8, M = 16\}$. For U = 8, there are

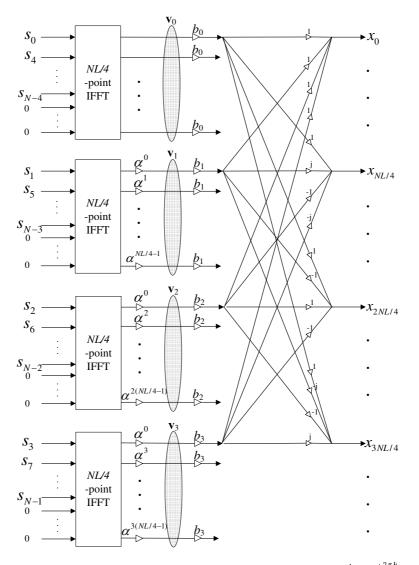


Figure 1: Proposed PTS approach for U=4, where the twiddle factor $\alpha^k = e^{j\frac{2\pi k}{NL}}$.

 4^6 independent sequences and we randomly pick 16 of the 4^6 sequences as our candidates for PAPR computation.

First let us look at the complexity of these methods. Table 1 and Table 2 show the complexity of different schemes for N = 64and N = 256 respectively. For the IFFT algorithm, we use the number of multiplications and additions listed in [9]. We have excluded all trivial multiplications (by $0, \pm 1, \pm j$) and also taken into consideration the fact that the sequences are zero-padded. From the tables, we see that the SLM method has the highest complexity and the proposed method has the lowest complexity. For the case of U = 4, we find that the number of multiplications in our method is about 1/3 of that in the original PTS method, and the number of additions is about 1/5 of the original PTS scheme. For the case of U = 8, our method still has a much lower complexity than the original PTS scheme with U = 4.

The performances of PAPR reduction of these method are shown in Fig. 2 and Fig. 3 for the cases of N=64 and N=256 respec-

tively. From the plots, we see that the SLM method has the best performance. For a CCDF of 10^{-3} , the performance of the original PTS scheme with $\{U = 4, M = 64\}$ is about 0.54dB worse than the SLM method for N = 64 and it is about 0.33dB worse than the SLM method for N = 256. The performance of the proposed method with $\{U = 4, M = 16\}$ is only 0.46dB worse than the original PTS scheme for N = 64 and it is only 0.38dB worse for N = 256. Notice that the number of independent candidates in the proposed method is M = 16 which is much smaller than that in the original PTS scheme (M = 64). Moreover, our method has a much lower computational complexity. Comparing the original PTS scheme with our method with $\{U = 8, M = 16\}$, we find that the two methods have a comparable PAPR performance but our method has a lower cost (see Tables 1 and 2). Also note from Fig. 2 and Fig. 3 that the method in [5] has a worse performance than our method even though it has a higher complexity.

Table 1: Complexity comparison, $N = 64$ and $L = 4$			
	MULs	ADDs	
SLM approach, $\{M = 16\}$	14336	24576	
Original PTS, $\{U = 4, M = 64\}$	1508	50176	
$[5], \{U = 4, M = 8\}$	1508	7168	
$[5], \{U = 8, M = 16\}$	2072	33536	
Proposed, $\{U = 4, M = 16\}$	515	9216	
Proposed, $\{U = 8, M = 16\}$	1220	13056	

Table 2: Complexity comparison, N = 256 and L = 4

	MULs	ADDs
SLM approach, $\{M = 16\}$	73728	131072
Original PTS, $\{U = 4, M = 64\}$	10020	200704
$[5], \{U = 4, M = 8\}$	10020	28672
$[5], \{U = 8, M = 16\}$	16520	134144
Proposed, $\{U = 4, M = 16\}$	3083	38912
Proposed, $\{U = 8, M = 16\}$	5376	54272

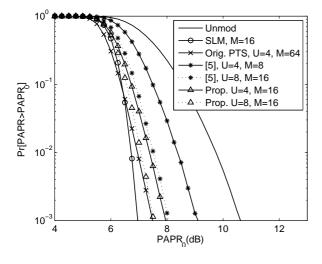


Figure 2: PAPR comparison, N=64 and the symbols are 16-QAM.

5. CONCLUSIONS

In this paper we have proposed a new way of partitioning the input symbol into disjoint subblocks in the PTS scheme. By using the DIT structure for IFFT, the complexity of IFFT computation can be significantly reduced. Moreover with new partition, the first two rotation factors can be set to 1 without affecting the PAPR reduction ability. Simulation results show that the PAPR of the proposed method is only slightly worse than the original PTS method but it needs a much smaller number of multiplications and additions.

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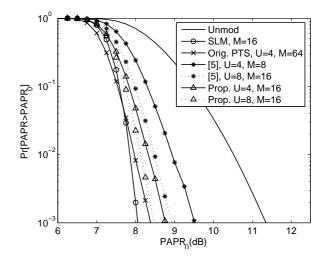


Figure 3: PAPR comparison, N=256 and the symbols are QPSK.

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