Filterbank Framework for Multicarrier Systems with Improved Subcarrier Separation

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Abstract— **Frequency separation among the subcarriers is of great importance for multicarrier systems. At the transmitter side better frequency separation leads to a smaller out-of-band spectral leakage and also less interference to radio frequency transmission. At the receiver side better separation gives more suppression of radio frequency interference. As these are frequency based characteristics, a filterbank representation renders a natural tool for formulating the problem. In this work, we will consider a filterbank framework for designing multicarrier systems with improved subcarrier separation.**

I. Introduction

The multicarrier systems have found important applications in DMT (discrete multitone) systems such as ADSL (asymmetric digital subscriber lines) [1] and VDSL (very high speed digital subscriber lines) [2]. In the conventional DFT based multicarrier system the transmitting and receiving filters come from rectangular windows, which are known to have large spectral sidelobes. The stopband attenuation is insufficient for many applications. For example, poor frequency separation at the transmitter side leads to significant spectral leakage. This could pose a problem in applications where the PSD (power spectral density) of the transmit signal is required to have a large roll-off in certain frequency bands. For example the PSD should also be attenuated in amateur radio bands to reduce interference or egress emission [2]. On the other hand poor frequency separation at the receiver side results in poor out-of-band rejection. In DMT applications such as ADSL and VDSL, some of the frequency bands are also used by radio transmission systems, whose signals can be coupled into the wires and this introduces radio frequency interference (RFI) or ingress. Ill frequency separation means many neighboring tones can be affected and the total transmission rate decreased.

Many methods have been proposed in the literature to improve the frequency characteristics of the transmitter and receiver. To improve the spectral roll-off of the transmit signal, a number of continuous-time pulse shaping filters [3], [4] and discrete-time windows [5], [6] have been proposed. More recently, windows have been proposed at the transmitter side for egress control [7], [8], [9] and at the receiver side for RFI suppression [10], [11], [12].

In this paper, we use a filterbank framework to design transmitters and receivers for better frequency separation. In particular we consider the inclusion of subfilters to improve the frequency selectivity of the transmitting and receiving filters while maintaining the orthogonality among the subcarriers. We will then design the transmitter for minimum spectral leakage and the receiver for maximum RFI suppression. Design examples will be given to show that transmitting/receiving filters with good spectral roll-off can be obtained.

II. Multicarrier Systems

The block diagram of a multicarrier system is as shown in Fig. 1. After proper time-domain equalization (if necessary), the channel is modeled as an FIR filter $C(z)$ of order L with additive noise $q(n)$. The input symbols $s_k(n)$ are passed through an M by M IDFT matrix $\mathbf{W}^\dagger$, where '$\dagger$' denotes conjugate transpose and $\mathbf{W}$ is the normalized $M \times M$ DFT matrix with $[\mathbf{W}]_{kn} = \frac{1}{\sqrt{M}} e^{-j\frac{2\pi kn}{M}}$, for $0 \le k, n \le M-1$. The outputs are converted to a block of M serial samples by the parallel to serial operation (P/S). Then a cyclic prefix of length ν is inserted. The length of the cyclic prefix ν is chosen so that $\nu \ge L$, which ensures that inter-block-interference (IBI) can be removed easily by discarding the prefix later. At the receiver, after prefix removal the samples are blocked into M by 1 vectors (S/P) for M-point DFT computation. The scalar multipliers $1/\lambda_k$ are called frequency domain equalizers with $\lambda_k = C_k$, where C_k are the M-point DFT of the channel impulse response c_n, namely $C_k = \sum_{n=0}^{L} c_n e^{-j2\pi kn/M}$, for $k = 0, 1, \cdots, M-1$. The transceiver is ISI free and the receiver is a zero-forcing receiver.

The system in Fig. 1 is known to have the filterbank representation in Fig. 2 [14]. The transmitting filter and receiving filter are given respectively by

$$F_k(z) = \frac{1}{\sqrt{M}} \sum_{i=0}^{N-1} e^{j(i-\nu)k\frac{2\pi}{M}} z^{-i}, \qquad (1)$$

$$H_k(z) = \frac{z^\nu}{\sqrt{M}} \sum_{i=0}^{M-1} e^{-j\frac{2\pi}{M}ik} z^i. \qquad (2)$$
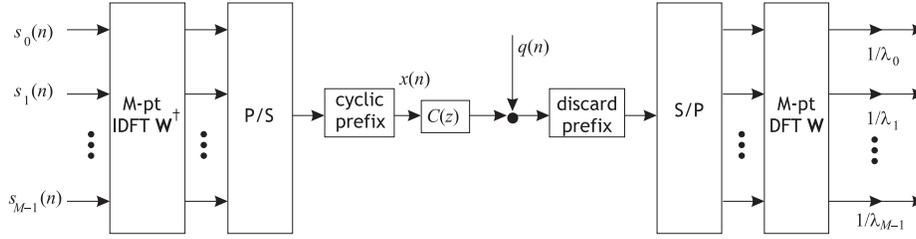
Fig. 1.   Block diagram of a multicarrier system.
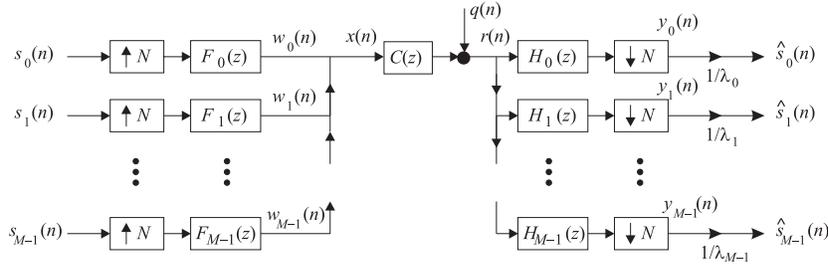
Fig. 2.   Filterbank representation of the multicarrier system in Fig. 1.

From (1) and (2), we see that the transmitting and receiving filters are derived from rectangular windows. In particular, the filter $F_0(z)$ is a rectangular window of length N. All the other transmitting filters are its scaled and frequency-shifted versions, namely, $F_k(z) = W^{\nu k} F_0(zW^k)$. Similarly, the first receiving filter $H_0(z)$ is also a rectangular window, but of length M. All the other receiving filters are its scaled and frequency-shifted versions, $H_k(z) = W^{-\nu k} H_0(zW^k)$. As the filters all come from rectangular windows, the frequency selectivity is not good.

III. TRANSCEIVER DESIGN WITH ZERO ISI

With the filterbank representation in Fig. 2, we can now obtain ISI free condition for the transceiver. Using the polyphase identity [13], we observe that the transfer function $T_{k,i}(z)$ from the i-th transmitter input $s_i(n)$ to the k-th signal $y_k(n)$ at the receiver (Fig. 2) is

$$T_{ki}(z) = [H_k(z)C(z)F_i(z)]_{\downarrow N}, \tag{3}$$

where the notation $[A(z)]_{\downarrow N}$ denotes the N-fold decimated version of $A(z)$. As the system has zero inter-block and inter-subcarrier ISI, and the subcarrier gain from the transmitter input $s_k(n)$ to the receiver output $\hat{s}_k(n)$ is one, we have $T_{ki}(z) = \lambda_k \delta(k - i)$. Summarizing, we can obtain the following lemma, which will be useful for later sections.

Lemma 1: Consider the system in Fig. 2. The transfer function $T_{k,i}(z)$ from the i-th transmitter input $s_i(n)$ to the k-th signal $y_k(n)$ at the receiver is given by

$$T_{ki}(z) = \lambda_k \delta(k - i), \quad 0 \le k, i \le M - 1. \tag{4}$$

The result holds for any FIR filter $C(z)$ of order $L \le \nu$. The constant λ_k are the M-point DFT of $C(z)$, i.e., $\lambda_k = C(z)|_{z=e^{j2\pi k/M}}$.

So long as the order of $C(z)$ is not larger than ν, the system is free from inter-block interference and inter-subcarrier interference. This means that, if we cascade another filter before or after the channel, as long as the product with $C(z)$ has order no larger than ν the overall system remains ISI free.

IV. RECEIVER DESIGN

To improve the frequency selectivity of the receiving filters, we introduce subfilters. Let the subfilters $Q_k(z)$ be FIR or order β. The k-th receiving filter is replaced by $H'_k(z) = H_k(z)Q_k(z)$. Now the transfer function from the i-th transmitter input $s_i(n)$ to the k-th signal $y_k(n)$ at the receiver (Fig. 2) becomes $T_{ki}(z) = [H_k(z)(Q_k(z)C(z))F_i(z)]_{\downarrow N}$; it has the same expression as (3) except that the channel is replaced by $Q_k(z)C(z)$. From the result in Lemma 1, we know the system is free from ISI as long as the order of $Q_k(z)C(z)$ satisfies $\beta + L \le \nu$. In particular, $T_{ki}(z)$ is the same as in (4) except that the FEQ coefficients λ_k are now the M-point DFT of the $Q_k(z)C(z)$. Let us choose the subfilters as shifted versions of the first subfilter, $Q_k(z) = Q_0(zW^k)$. Then the new receiving filter becomes $H'_k(z) = W^{-\nu k}H'_0(zW^k)$. They are also shifted versions of $H'_0(z)$ except for a scalar. These receiving filters again form a DFT bank and thus can be implemented efficiently [13].

We can maximize the frequency separation among the

subcarriers by minimizing

$$\phi = \int_{\omega \in O} |H_0'(e^{j\omega})|^2 d\omega, \qquad (5)$$

where O denotes the stopband. As $H_0'(z)$ is the product of $Q_0(z)$ and $H_0(z)$, it can be written as $H_0'(e^{j\omega}) = \boldsymbol{\tau}(\omega)\mathbf{q}$, where $\mathbf{q}$ consists of coefficients of $Q_0(z)$ and $\boldsymbol{\tau}(\omega) = H_0(e^{j\omega}) \begin{pmatrix} 1 & e^{-j\omega} & \cdots & e^{-j\beta\omega} \end{pmatrix}$. Therefore, we can write the objective function ϕ in (5) as

$$\phi = \mathbf{q}^\dagger \mathbf{B} \mathbf{q}, \text{ where } \mathbf{B} = \int_{\omega \in O} \boldsymbol{\tau}^\dagger(\omega)\boldsymbol{\tau}(\omega)d\omega. \qquad (6)$$

The matrix $\mathbf{B}$ is positive definite because ϕ represents the stopband energy of the receiving $F_0'(z)$. Thus ϕ is always positive as $F_0'(z)$ is FIR. To minimize ϕ, we can choose $\mathbf{q}$ as the eigen vector associated with the smallest eigen value of $\mathbf{B}$.

Example 1. RFI reduction. In this example, we show RFI can be reduced by due to improved subcarrier frequency separation. The DFT size is $M = 512$ and cyclic prefix length is $\nu = 40$. The order of the subfilters is $\beta = 10$. The channel used in this example is VDSL loop#1 (4500ft) [2] and the channel noise is AWGN of -140 dBm. A time domain equalizer [16] is used to shorten the channel. Model 1 differential mode RFI interference of -55dBm is considered [2]. There are 4 RFI sources in the simulations, 660, 710, 770 and 1050 KHz.

The signal-to-interference-noise ratio (SINR) of the subcarriers (tones) are as shown in Fig. 3. For comparison, we have also shown the subcarriers SINRs for the cases of rectangular, Hanning and Blackman windows. The receiver with subfilters enjoys higher SINR for the tones that are close to the RFI frequencies. As a result, a higher transmission rate can be achieved. The corresponding transmission rate is 7.44 Mbits/sec, while the transmission rates for the cases of rectangular, Hanning and Blackman windows are 6.84, 7.16 and 7.16 Mbits/sec, respectively.
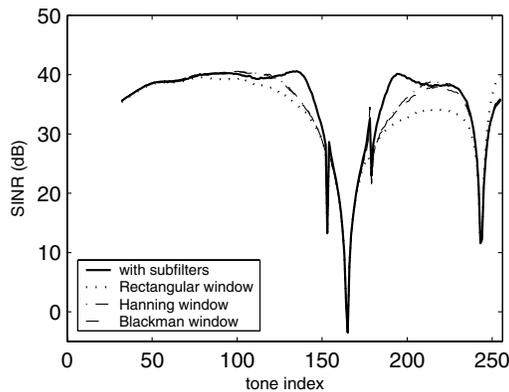
Fig. 3. Receiver with subfilters for RFI reduction: the subcarrier SINRs.

V. TRANSMITTER DESIGN

Similar to the case of the receiving end, we can also introduce subfilters to the transmitter side to improve the frequency selectivity of the transmitting filters. Suppose the subfilters are $P_k(z)$, of order α. The k-th transmitting filter is replaced by $F_k'(z) = F_k(z)P_k(z)$. Now the transfer function from the i-th transmitter input $s_i(n)$ to the k-th signal $y_k(n)$ at the receiver (Fig. 2) becomes $T_{ki}(z) = [H_k(z)(P_i(z)C(z))F_i(z)]_{\downarrow N}$. Similarly using the result in Lemma 1, we know the overall system remains ISI free as long as the order of the subfilters α satisfy $\alpha + L \leq \nu$. Also, the transfer function $T_{ki}(z)$ is the same as in (4), except now λ_k are now the M-point DFT of $P_k(z)C(z)$. The new receiving filters $F_k'(z)$ are of length $N + \alpha$, as $F_k(z)$ are of length N. As in the receiver case, we choose the subfilters as shifted versions of the first subfilter, i.e., $P_k(z) = P_0(zW^k)$. The resulting transmitting filters form a DFT bank, which can be implemented efficiently [13].

When the transmitting filters are shifted versions of one prototype, the spectrum of the transmitted signal consists of shifts of $|F_0'(e^{j\omega})|^2$ [15]. The spectral leakage can be minimized by minimizing the stopband energy of the prototype filter $F_0'(z)$. Let $\boldsymbol{\tau}(\omega) = F_0(e^{j\omega}) \begin{pmatrix} 1 & e^{-j\omega} & \cdots & e^{-j\alpha\omega} \end{pmatrix}$. Similar to the case of receiver design, the stopband energy of the prototype filter $F_0'(z)$ can be written in the form

$$\mathcal{S} = \mathbf{p}^\dagger \mathbf{A} \mathbf{p}, \text{ where } \mathbf{A} = \int_{\omega \in O} \boldsymbol{\tau}^\dagger(\omega)\boldsymbol{\tau}(\omega)d\omega. \qquad (7)$$

Therefore the spectral leakage can be minimized by choosing $\mathbf{p}$ to be the eigen vector associated with the minimum eigen value of $\mathbf{A}$.

Example 2. Spectral leakage suppression. In this example, we design subfilters at the transmitter for spectral leakage suppression. The block size $M = 512$ and prefix length $\nu = 40$. The order α of the subfilters is 20. Fig. 4 shows the spectrum of the transmitter output. The subcarriers used are 38 to 99 and 111 to 255 as in [10]. The subcarriers with indices smaller than 38 are reserved for voice band and upstream transmission, and those with indices between 99 and 111 are for egress control. We see that the spectrum of the windowed output has a much smaller spectral leakage in unused bands.

VI. CONCLUSIONS

In this paper, we use a filterbank framework to design multicarrier system for better subcarrier separation. Transmitting and receiving filters with better frequency selectivity can be obtained. Design examples are given to demonstrate the usefulness of the proposed design. Improved subcarrier separation at the transmitter allows the transmitted spectrum to have a faster spectral roll-off. Also better subcarrier separation at the receiver yields more suppression of RFI noise.
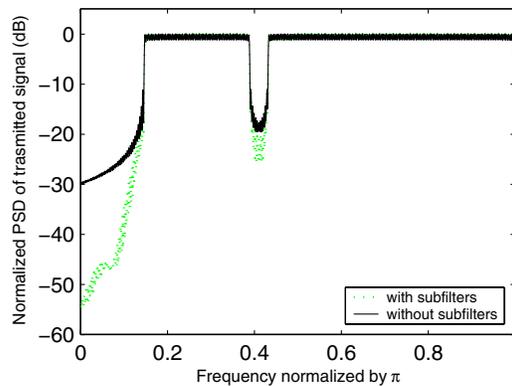
Fig. 4. Transmitter with subfilters for spectral leakage suppression: power spectrum of the transmitted signal.

REFERENCES

[1] *Asymmetric Digital Subscriber Lines (ADSL)-Metallic Interface*, ANSI T1.413, 1998.

[2] *Very-high Speed Digital Subscriber Lines (VDSL)-Metallic Interface*, ANSI T1.424, 2002.

[3] A. Vahlin and N. Holte, *Optimal Finite Duration Pulses for OFDM*, *IEEE Trans. Communications*, vol. 44, no. 1, pp. 10-14, Jan. 1996.

[4] M. Pauli and P. Kuchenbecker, *On the reduction of the out-of-band radiation of OFDM-signals*, Proc. IEEE International Conference on Communications, vol. 3, pp. 1304-1308, June 1998.

[5] H. Boelcskei, P. Duhamel, and R. Hleiss, *Design of pulse shaping OFDM/OQAM systems for high data-rate transmission over wireless channels*, Proc. IEEE International Conference on Communications, 1999.

[6] P. Siohan, C. Siclet, and N. Lacaille, *Analysis and Design of OFDM/OQAM Systems Based on Filterbank Theory*, IEEE Trans. Signal Processing, May 2002.

[7] G. Cuypers, K. Vanbleu, G. Ysebaert, M. Moonen *Egress Reduction By Intra-symbol Windowing in DMT-based Transmissions*, Proc. IEEE International Conference on Acoustics, Speech and Signal Processing, 2003.

[8] Chun-Yang Chen, and See-May Phoong, *Per tone Shaping filters for DMT Transmitters*, Proc. IEEE Int. Conf. Acoustic, Speech, and Signal Processing, May 2004.

[9] Yuan-Pei Lin and See-May Phoong, *Window Designs for DFT based Multicarrier System*, IEEE Trans. Signal Processing, March 2005.

[10] G. Cuypers, K. Vanbleu, G. Ysebaert, M. Moonen, and P. Vandaele, *Combining raised cosine windowing and per tone equalization for RFI mitigation in DMT receivers*, Proc. IEEE International Conference on Communications, 2003.

[11] A. J. Redfern, *Receiver window design for multicarrier communication systems*, IEEE Journal on Selected Areas in Communications, vol. 20, no. 5, June 2002.

[12] G. Ysebaert, K. Vanbleu, G. Cuypers, and M. Moonen, *Joint Window and Time Domain Equalizer Design for Bit Rate Maximization in DMT-Receivers*, IEEE Journal on Selected Areas in Communications, vol. 20, no. 5, June 2002.

[13] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Englewood Cliffs,Prentice-Hall, 1993.

[14] Yuan-Pei Lin and See-May Phoong, *OFDM Transmitters: Analog Representation and DFT Based Implementation*, IEEE Trans. Signal Processing, Sep. 2003.

[15] Yuan-Pei Lin, Chien-Chang Li, and See-May Phoong, *A Filterbank Approach to Window Design for Multicarrier Systems*, in preparation.

[16] R. K. Martin, J. Balakrishnan, W. A. Sethares, and C. R. Johnson Jr., *A Blind, Adaptive TEQ for Multicarrier Systems*, IEEE Signal Processing Letters, vol. 9, no. 11, Nov. 2002.