BER OPTIMIZED CHANNEL INDEPENDENT PRECODER FOR OFDM SYSTEM

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Abstract- **We consider the minimization of bit error rate (BER) for OFDM systems with orthogonal precoders. For low SNR, we show that the conventional OFDM system is the optimal solution and the optimal precoder is the identity matrix. For high SNR, corresponding to a relatively practical range of BER, there are channel independent solutions for optimal precoders. One such solution is the DFT matrix and the resulting optimal transceiver becomes the single carrier transmission system. Furthermore, in this case the conventional OFDM system has the largest BER among all OFDM systems with orthogonal precoders.**

1. INTRODUCTION

The OFDM (orthogonal frequency division multiplexing) transceiver has found applications in a wide range of wireless transmission channels, [1]-[2]. The block based transmitter and receiver perform respectively M-point IDFT and DFT computation, where M is the number of tones or number of subchannels. As a result, an FIR channel is converted into M frequency non-selective parallel subchannels. The subchannel gains are the M-point DFT of the FIR channel impulse response. The channel dependent part of the transceiver is a set of M scalars at the receiver and the transmitter is channel independent. This feature is particularly attractive for wireless applications, where the transmitter usually does not have knowledge of the channel. Due to the lack of the channel profile at the transmitter of wireless transmission, bit and power allocation are generally not employed.

The single carrier system [3] is also a DFT based transceiver with a channel independent transmitting matrix, the identity matrix. The receiver performs both DFT and IDFT operations. It can be viewed as the OFDM system with a DFT precoder. It is demonstrated that the single carrier system has a very low PAPR (peak to average power ratio). Furthermore, it outperforms the OFDM system for useful range of bit error rate (BER). However optimality of the single carrier system has not been addressed.

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Design of more general block transceivers optimal in the sense of minimum transmission power or minimum total noise power has been of great interest. In [4], optimal block transceivers for minimum noise power are given in terms of the channel and noise power spectrum. In [5], optimal block transceivers with optimal bit loading are designed for minimizing transmission power. In these systems, the channel profile is required for designing the transmitter.

In this paper, we will consider the minimization of BER for the class of OFDM transceivers with orthogonal precoders. The underlying class is in fact the class of block transceivers with orthogonal transmitters. We will address the design of optimal precoders with the assumption that there is no bit and power allocation at the transmitter. Notice that the objective is bit error rate, not mean square error as is in most of the earlier results. For low SNR, we show that the conventional OFDM system is the optimal solution and the optimal precoder is the identity matrix. For higher SNR associated with a useful range of BER we can design optimal precoders that have channel independent solutions. Two types of channel independent precoders will be given. One is the DFT matrix and the other one is the Hadamard matrix. In the former case, the system becomes the single carrier system [3]. We will derive the results for BPSK modulations but generalizations to PAM, PSK, and QAM can be obtained with slight modifications. Furthermore we will show that for the range of bit error rates commonly considered in practice, the conventional OFDM system has the worst BER among all block transceivers with orthogonal transmitters.

2. OFDM WITH ORTHOGONAL PRECODERS

The block diagram of the conventional OFDM system is as shown in Fig. 1. The transmitter performs IDFT and the receivers performs DFT computation. At the receiver the quantities P_k are the M-point DFT of the channel impulse response $p(n)$. We assume that the length of the cyclic prefix is no shorter than the channel impulse response, so the transceiver is ISI free or zero-forcing.

Fig. 2 shows the block diagram of the OFDM system with a precoding matrix $\mathbf{T}$ at the transmitter, where $\mathbf{T}$ is

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orthogonal with $\mathbf{T}^\dagger\mathbf{T} = \mathbf{I}$. To have a zero-forcing receiver, $\mathbf{T}^\dagger$ is cascaded to the end of the receiver. This is also a general block transceiver with an orthogonal transmitting matrix $\mathbf{G} = \mathbf{W}^\dagger\mathbf{T}$. The resulting transmitting matrix $\mathbf{G}$ and receiving matrix $\mathbf{S}$ are as shown in Fig. 2.

Bit Error Rate. We assume that the channel noise $\nu(n)$ is complex AWGN with power spectral density $2N_0$ and the modulation scheme is BPSK, modulation symbols $s_k = \pm\sqrt{\mathcal{E}_s}$. Let the noise vector be $\mathbf{e} = \mathbf{x} - \mathbf{s}$. As BPSK symbols are real the relevant noise variance $\sigma^2(i)$ of the i-th subchannel is given by

$$\sigma^2(i) = N_0 \sum_{k=0}^{M-1} \frac{|t_{k,i}|^2}{|P_k|^2}, \quad i = 0, 1, \cdots, M-1, \quad (1)$$

where $t_{k,i}$ denotes the (k,i)-th element of $\mathbf{T}$. Let $\beta(i) = \sigma^2(i)/\mathcal{E}_s$, the noise-to-signal ratio (NSR) of i-th subchannel, then

$$\beta(i) = \frac{1}{\gamma} \sum_{k=0}^{M-1} \frac{|t_{k,i}|^2}{|P_k|^2}, \quad \gamma = \mathcal{E}_s/N_0. \quad (2)$$

As $\mathbf{T}$ is orthogonal, we have $\sum_{i=0}^{M-1} |t_{k,i}|^2 = 1$. Using this fact, we can write the average mean squared error $\mathcal{E}_{rr} = 1/M \sum_{i=0}^{M-1} \sigma^2(i)$ as

$$\mathcal{E}_{rr} = \frac{N_0}{M} \sum_{i=0}^{M-1} \frac{1}{|P_i|^2}. \quad (3)$$

The average mean squared error is independent of $\mathbf{T}$. All OFDM transceivers with an orthogonal precoder $\mathbf{T}$ has the same $\mathcal{E}_{rr}$.

For BPSK modulation, the BER of the i-th subchannel is $\mathcal{P}_T(i) = Q\left(\sqrt{\mathcal{E}_s/\sigma^2(i)}\right)$. For the convenience of subsequent discussion, we introduce the function

$$f(y) = Q(1/\sqrt{y}). \quad (4)$$

In terms of $f(\cdot)$ and subchannel NSR, we have

$$\mathcal{P}_T(i) = f(\beta(i)), \quad \mathcal{P}_T = \frac{1}{M} \sum_{i=0}^{M-1} f(\beta(i)),$$

where $\mathcal{P}_T$ is the average bit error rate.

- *The OFDM case:* The orthogonal matrix $\mathbf{T} = \mathbf{I}$. We have,

  $$\sigma^2_{ofdm}(i) = N_0/|P_i|^2, \quad i = 0, 1, \cdots, M-1. \quad (5)$$

  For the i-th subchannel, the NSR $\beta_{ofdm}(i)$ is

  $$\beta_{ofdm}(i) = \frac{1}{\gamma|P_i|^2},$$

  where γ is the signal-to-noise ratio $\mathcal{E}_s/N_0$. The BER of the OFDM system becomes,

  $$\mathcal{P}_{ofdm} = \frac{1}{M} \sum_{i=0}^{M-1} f(\beta_{ofdm}(i)) = \frac{1}{M} \sum_{i=0}^{M-1} f\left(\frac{1}{\gamma|P_i|^2}\right).$$

- *The single carrier case:* The orthogonal matrix $\mathrm{T} = \mathbf{W}$. The noise variances in all the subchannels are the same and it is equal to the average mean squared error $\sigma^2_{sc} = \mathcal{E}_{rr}$. As a result, all the subchannels have the same NSR $\beta_{sc} = \mathcal{E}_{rr}/\mathcal{E}_s$ or

  $$\beta_{sc} = \frac{1}{M} \sum_{i=0}^{M-1} \frac{1}{\gamma|P_i|^2},$$

  which is also the average of $\beta_{ofdm}(k)$. In fact, as the average mean squared error is always $\mathcal{E}_{rr}$, we have $\beta_{sc} = 1/M \sum_{i=0}^{M-1} \beta(i)$ for any orthogonal T. The bit error rate of the single carrier system is

  $$\mathcal{P}_{sc} = f(\beta_{sc}) = f\left(\frac{1}{M} \sum_{i=0}^{M-1} \frac{1}{\gamma|P_i|^2}\right). \quad (6)$$

Notice that $\mathbf{T}$ is orthogonal with $\mathbf{T}^\dagger\mathbf{T} = \mathbf{I}$ and the columns of T has unit energy, i.e., $\sum_{i=0}^{M-1} |t_{i,k}|^2 = 1$, for all k. It can be verified that,

$$\min_k \sigma^2_{ofdm}(k) \leq \sigma^2(i) \leq \max_k \sigma^2_{ofdm}(k), \forall i. \quad (7)$$

The relations hold for any orthogonal precoder T. For different choice of $\mathbf{T}$, the redistributed noise variances are bounded in between $\min_k \sigma^2_{ofdm}(k)$ and $\max_k \sigma^2_{ofdm}(k)$. For any orthogonal precoder $\mathbf{T}$, the best subchannel is no better than the best subchannel of the OFDM system and the worst subchannel is no worse than the worst subchannel of the OFDM system. In the next section, we derive the optimal $\mathbf{T}$ such that BER is minimized.

3. OPTIMAL T

As we will see later, the bit error rate performance is closely related to the behavior of the function $f(\cdot)$ to be given in the following lemma [6].

Lemma 1 *The function $f(y) = Q(\frac{1}{\sqrt{y}})$ is monotone increasing. It is convex when $y \leq 1/3$ and concave when $y > 1/3$.*

A plot of $f(y)$ is shown in Fig. 3. For a given channel, the subchannels are operating in the convex or concave region of the function $f(\cdot)$ depending on the SNR $\gamma = \mathcal{E}_s/N_0$. We define three useful SNR quantities,

$$\gamma_0 = \min_i \frac{3}{|P_i|^2}, \quad \bar{\gamma} = \frac{1}{M} \sum_{i=0}^{M-1} \frac{3}{|P_i|^2} \quad \gamma_1 = \max_i \frac{3}{|P_i|^2}.$$

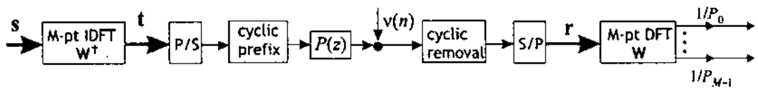

Figure 1: The block diagram of the OFDM system over a channel $P(z)$ with additive noise $\nu(n)$.

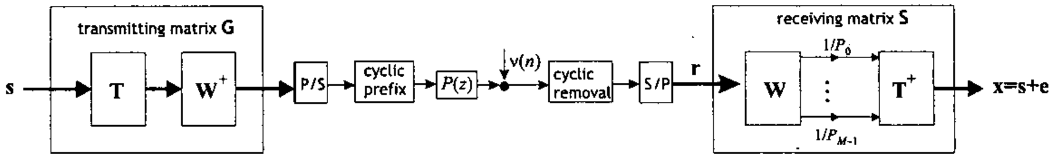

Figure 2: The OFDM system with a precoder $\mathbf{T}$.

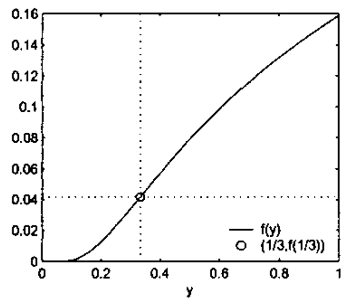

Figure 3: Plot of $f(y) = Q(1/\sqrt{y})$ for $0 \le y \le 1$.

By definition they satisfy $\gamma_0 \le \bar{\gamma} \le \gamma_1$. When $\gamma = \bar{\gamma}$, we have $\beta_{sc} = \frac{1}{3}$. The value $\bar{\gamma}$ is the SNR for which the subchannels of the single carrier system are operating on the boundary between convex and concave region of $f(\cdot)$. Notice that if $\gamma \ge \gamma_1$, the subchannel NSR $\beta_{ofdm}(i) \le \frac{1}{3}$, for all i; $\beta_{ofdm}(i)$ falls into the convex region of the function $f(y)$. On the other hand, if $\gamma \le \gamma_0$, all the subchannel NSRs fall into the concave region of $f(y)$. For these two ranges of γ, we can establish the following relation among the BER performance of the OFDM system, the single carrier system and the OFDM system with an arbitrary orthogonal precoder $\mathbf{T}$.

Theorem 1*Let $\mathcal{P}_T$ be the bit error rate of the OFDM systems with an orthogonal precoder $\mathbf{T}$ in Fig. 2. Then*

$$\mathcal{P}_{ofdm} \ge \mathcal{P}_T \ge \mathcal{P}_{sc}, \quad \text{for } \gamma \ge \gamma_1,$$

$$\mathcal{P}_{ofdm} \le \mathcal{P}_T \le \mathcal{P}_{sc}, \quad \text{for } \gamma \le \gamma_0.$$

Each of the two inequalities relating $\mathcal{P}_{ofdm}$ and $\mathcal{P}_T$ becomes an equality if and only if $|P_0| = |P_1| = \cdots = |P_{M-1}|$. Each of the two inequalities relating $\mathcal{P}_T$ and $\mathcal{P}_{sc}$ becomes an equality if and only if subchannel noise variances $\sigma(i)$ are equalized, $\sigma(i) = \mathcal{E}_{rr}/M$, where $\mathcal{E}_{rr}$ is as given in (3).

The proof is given in the Appendix.

The results in the above theorem imply that, among all block transceivers with orthogonal transmitters the OFDM system is the optimal solution for $\gamma \le \gamma_0$; when all the subchannels are operating in the concave region of $f(\cdot)$, the OFDM system has the smallest error rate. For $\gamma \ge \gamma_1$ it is the worst solution; when all the subchannels are operating in the convex region of $f(\cdot)$, the OFDM system has the largest error rate. However,as we will see later, the former case $\gamma \le \gamma_0$ corresponds to a high error rate that is of little interest in many applications. The later case ($\gamma \ge \gamma_1$) corresponds to a more practical range of BER. The error rate behavior can be analyzed by considering the value of γ in the following different regions:

(1) The case $\gamma \le \gamma_0$: In this range, the OFDM system is the optimal solution. All the subchannels have $\beta_{ofdm}(k) \ge 1/3$ and hence $\mathcal{P}_{ofdm} \ge f(1/3) = 0.0416$. In this range of SNR the error rate $\mathcal{P}_{ofdm}$ is at least 0.0416, a BER too large for many applications. Furthermore the minimum BER 0.0416 can be achieved only when all the subchannels have BER $= 0.0416$, which requires $|P_0| = |P_1| = \cdots = |P_{M-1}|$.

(2) The case $\gamma \ge \gamma_1$: For this range, the OFDM system has the largest BER and the BERs of all orthogonal precoders are lower bounded by $\mathcal{P}_{sc}$. All the subchannels are operating in the convex region of $f(\cdot)$ and $\beta(k) \le 1/3$. The subchannel error rate $\mathcal{P}_T(k) \le f(1/3) = 0.0416$ and the average $\mathcal{P}_T \le 0.0416$. Notice that, when $\gamma = \gamma_1$,the worst subchannel of the OFDM system has error rate $Q(\sqrt{3}) = 0.0416$ and the average BER is at least $Q(\sqrt{3})/M$. So γ_1 is also the minimum SNR to have error rate lower than $Q(\sqrt{3})/M$ in the OFDM system. For example, for $M = 16$, γ_1 is the smallest SNR for achieving bit error rate $Q(\sqrt{3})/16 = 0.0026$ and for $M = 64$, γ_1 is the smallest SNR for achiev-

ing bit error rate $Q(\sqrt{3})/64 = 6.4 \times 10^{-4}$. The case SNR $\gamma \geq \gamma_1$ corresponds to a more useful range of BER.

(3) The case $\gamma_0 \leq \gamma \leq \gamma_1$: For this range, $\mathcal{P}_{ofdm}$ is not necessarily larger or smaller than $\mathcal{P}_{sc}$. We can plot $\mathcal{P}_{sc}$ and $\mathcal{P}_{ofdm}$ as functions of γ. In most of our experiments, the crossing of the two curves happens at an SNR around $\bar{\gamma}$, i.e., the SNR for which β_{sc} falls in the convex region of the function $f(\cdot)$.

Channel Independent Transmitters Achieving $\mathcal{P}_{sc}$

Theorem 1 states that, we have $\mathcal{P}_T = \mathcal{P}_{sc}$ if $\sigma^2(i)$ are equalized, i.e., $N_0 \sum_{k=0}^{M-1} |t_{k,i}|^2/|P_k|^2 = \mathcal{E}_{rr}$, where $\mathcal{E}_{rr}$ as given in (3). In particular, for channel independent solutions of $\mathbf{T}$,we can choose

$$|t_{m,n}| = \frac{1}{\sqrt{M}}, \quad 0 \leq m, n \leq M-1. \tag{8}$$

In this case all the subchannel BERs are the same, $\mathcal{P}_T(k) = \mathcal{P}_T = \mathcal{P}_{sc}$. When M is a power of 2, it is known [7]that there is a class of matrices satisfying (8). Two well-known solutions are the DFT matrix $\mathbf{W}$ and the Hadamard matrix $\mathbf{H}$. When $\mathbf{T} = \mathbf{W}$, the transceiver in Fig. 2 becomes the single carrier system in [3]. The Hadamard matrices can be generated recursively. The 2×2 Hadamard matrix is given by

$$\mathbf{H}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

The $2n \times 2n$ Hardamard matrix is given in terms of the $n \times n$ Hardamard matrix by

$$\mathbf{H}_{2n} = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{H}_n & \mathbf{H}_n \\ \mathbf{H}_n & -\mathbf{H}_n \end{pmatrix}.$$

The Hadamard matrix is real with elements equal to ± 1. The resulting transmitting matrix $\mathbf{G} = \mathbf{W}^\dagger \mathbf{H}$ will be complex. The implementation of Hadamard matrices requires only additions. The complexity of the transceiver is slightly more than the OFDM system due to the two extra Hadamard matrices.

Remarks. The derivations in this paper are carried out for BPSK modulation. The results will be valid for PAM, QAM, and PSK with slight modifications. In these cases, the subchannel bit error rate can be expressed $\alpha f(c\sigma^2(k)/\mathcal{E}_s)$, for some constants α and c independent of the subchannels. For example, for PAM modulation with each symbol carrying b bits, the subchannel symbol error rate is given by $2(1-2^{-b})Q\left(\sqrt{\frac{3\mathcal{E}_s}{(2^{2b}-1)\sigma^2(k)}}\right)$. The expression has the form $\alpha f(c\frac{\sigma^2(k)}{\mathcal{E}_s})$, where $\alpha = 2(1-2^{-b})$ and $c = (2^{2b}-1)/3$. When SNR γ is small enough such that the subchannels of the OFDM system are operating in the concave region of $f(\cdot)$, the OFDM system is the optimal solution. We can verify that the condition for this is $\gamma \leq \gamma_0$,whe re γ_0 now is

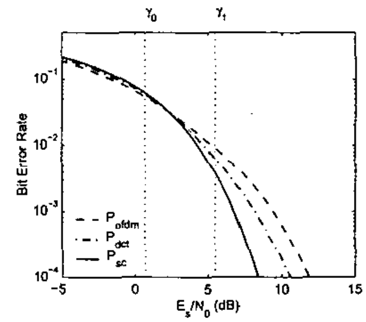

Figure 4: Example 1. Performance comparison of $\mathcal{P}_{ofdm}$, $\mathcal{P}_{dct}$ and $\mathcal{P}_{sc}$ systems for the channel $P(z)$.

$\min_k \frac{3c}{|P_k|^2}$. Similarly, when the subchannels of the OFDM system are operating in the convex region of $f(\cdot)$,the minimum bit error rate is $\mathcal{P}_{sc}$. The condition for this is $\gamma \geq \gamma_1$, where γ_1 now is $\max_k \frac{3c}{|P_k|^2}$.

4. EXAMPLES

We will assume that the noise is AWGN with variance $2N_0$. The modulation symbols are BPSK with values equal to $\pm\sqrt{\mathcal{E}_s}$ and SNR $\gamma = \mathcal{E}_s/N_0$. The number of subchannels M is 64. The length of cyclic prefix is 3.

Example 1. The channel $P(z)$ used in this example has 4 coefficients ($L = 3$), $-1.152 - j1.071$, $0.457 - j0.286$, $0.145 - j0.129$, $-0.0546 - j0.149$. We compute $\max_i |P_i|^2$ and $\min_i |P_i|^2$ respectively as 4.34 and 0.55; $\gamma_0 = \min_i 2/|P_i|^2$ is 0.69 dB and $\gamma_1 = \max_i 2/|P_i|^2$ is 5.49 dB.

Fig. 4 shows $\mathcal{P}_{ofdm}$ and $\mathcal{P}_{sc}$ for different SNR γ. In the same plot, we also show the BER of the block transceiver when the transmitting matrix $\mathbf{G}$ is an orthogonal DCT matrix, denoted as $\mathcal{P}_{dct}$. In this case, the precoder $\mathbf{T}$ given by $\mathbf{WG}$ does not have the unit magnitude property in (8). Whenever SNR $\gamma = \mathcal{E}_s/N_0$ is larger than $\gamma_1 = 5.49$ dB, $\mathcal{P}_{sc}$ given in (6) becomes the minimum BER for any orthogonal precoder $\mathbf{T}$. For $\gamma = 5.49$ dB, the corresponding $\mathcal{P}_{sc} \approx 0.004$. In other words, for BER ≤ 0.004, the single carrier system is the optimal solution among all OFDM systems with orthogonal precoders. For $\gamma \leq \gamma_0$, the conventional OFDM system is the optimal solution. For $\gamma = \gamma_0$, the BER is $\mathcal{P}_{ofdm} \approx 0.05$. For either SNR range, $\gamma \leq \gamma_0$ or $\gamma \geq \gamma_1$, the performance of $\mathcal{P}_{dct}$ is in between $\mathcal{P}_{ofdm}$ and $\mathcal{P}_{sc}$ when $\gamma \geq \gamma_1$.

Example 2. We use a multipath fading channel with 4 coefficients. The coefficients are obtained from independent complex Gaussian random variables with zero mean and variances given respectively by $8/15$, $4/15$, $2/15$, and

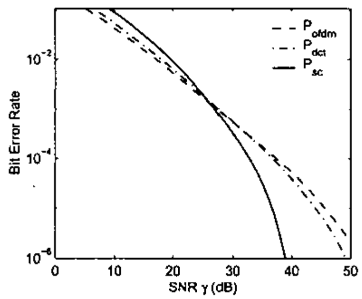

Figure 5: Example 2. The performance of $\mathcal{P}_{ofdm}$, $\mathcal{P}_{dct}$ and $\mathcal{P}_{sc}$ over a 4-tap multipath fading channel.

$1/15$. The BER performance of $\mathcal{P}_{ofdm}$, $\mathcal{P}_{dct}$ and $\mathcal{P}_{sc}$ are shown in Fig. 5. For large SNR, and a correspondingly more useful BER range, $\mathcal{P}_{sc}$ requires a significantly smaller transmission power than $\mathcal{P}_{ofdm}$ for the same BER. For example, for BER equal to 10^{-4}, the required SNR for $\mathcal{P}_{sc}$ is approximately 4.5 dB less than that for $\mathcal{P}_{ofdm}$. The performance of $\mathcal{P}_{dct}$ is in between $\mathcal{P}_{sc}$ and $\mathcal{P}_{ofdm}$ in low SNR and high SNR ranges.

APPENDIX

Given a set of numbers, $y_0, y_1, \cdots, y_{M-1}$ with $0 \le y_i \le 1/3$, the strictly monotone increasing property and the convexity of $f(y)$ implies

$$\sum_{i=0}^{M-1} \lambda_i f(y_i) \ge f\left(\sum_{i=0}^{M-1} \lambda_i y_i\right), \text{ where } \lambda_i \ge 0, \sum_{i=0}^{M-1} \lambda_i = 1.$$

The equality holds if and only if $y_0 = y_1 = \cdots = y_{M-1}$. Similarly, given $y_0, y_1, \cdots, y_{M-1}$ with $y_i \ge 1/3$, the concave property of $f(y)$ for $y \ge 1/3$ implies

$$\sum_{i=0}^{M-1} \lambda_i f(y_i) \le f\left(\sum_{i=0}^{M-1} \lambda_i y_i\right), \text{ where } \lambda_i \ge 0, \sum_{i=0}^{M-1} \lambda_i = 1.$$

The equality holds if and only if $y_0 = y_1 = \cdots = y_{M-1}$.

Let us first consider the case $\gamma \ge \gamma_1$. For this range, the subchannel NSR of the OFDM system $\beta_{ofdm}(i) \le 1/3$. Using (7), the subchannel NSR for 1 general orthogonal precoder $\mathbf{T}$ also satisfies $\beta(i) \le 1/3$ and it is in the convex region of $f(y)$. We have,

$$\mathcal{P}_T = \frac{1}{M}\sum_{i=0}^{M-1} f(\beta(i)) \ge f\left(\frac{1}{M}\sum_{i=0}^{M-1}\beta(i)\right) = \mathcal{P}_{sc}. \quad (9)$$

On the other hand, using (2), we have

$$f(\beta(i)) = f\left(\sum_{k=0}^{M-1} |t_{k,i}|^2 \frac{1}{\gamma|P_k|^2}\right) \le \sum_{k=0}^{M-1} |t_{k,i}|^2 f\left(\frac{1}{\gamma|P_k|^2}\right).$$

The inequality follows from the fact that $1/(\gamma|P_k|^2)$ is in the convex region of $f(\cdot)$ for $\gamma \ge \gamma_1$. Therefore,

$$\begin{aligned}\mathcal{P}_T &= \frac{1}{M}\sum_{i=0}^{M-1} f(\beta(i)) \\ &= \frac{1}{M}\sum_{i=0}^{M-1} f\left(\sum_{k=0}^{M-1} |t_{k,i}|^2 \frac{1}{\gamma|P_k|^2}\right) \\ &\le \frac{1}{M}\sum_{k=0}^{M-1}\sum_{i=0}^{M-1} |t_{k,i}|^2 f\left(\frac{1}{\gamma|P_k|^2}\right) \\ &= \frac{1}{M}\sum_{k=0}^{M-1} f\left(\frac{1}{\gamma|P_k|^2}\right) = \mathcal{P}_{ofdm},\end{aligned} \quad (10)$$

where we have used the fact that, for any orthogonal $\mathbf{T}$, its rows have unit energy, $\sum_{i=0}^{M-1} |t_{k,i}|^2 = 1$ for all k. Combining (9) and (10), we obtain $\mathcal{P}_{ofdm} \ge \mathcal{P}_T \ge \mathcal{P}_{sc}$ for $\gamma \ge \gamma_1$. Similarly, when $\gamma \le \gamma_0$, we can show that $\mathcal{P}_{ofdm} \le \mathcal{P}_T \le \mathcal{P}_{sc}$.

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5. REFERENCES

[1] ISO/IEC, IEEE Std. 802.11a, 1999.

[2] ETSI, "Digital Audio Broadcasting (DAB) to Mobile, Protable and Fixed Receivers," ETS 300 401, 1994.

[3] H. Sari, G. Karam and I. Jeanclaude, "Frequency-Domain Equalization of Mobile Radio and Terrestrial Broadcast Channels," Globecom, San Francisco, 1994.

[4] A. Scaglione, G. B. Giannakis, and S. Barbarossa "Redundant FilterBank Precoders and Equalizers Part I: Unification and Optimal Designs," *IEEE Trans. Signal Processing,* v ol. 47, no. 7, July 1999.

[5] Yuan-Pei Lin and See-May Phoong, "Optimal ISI Free DMT Transceivers for Distorted Channels with Colored Noise," *IEEE Trans. Signal Processing*, vol. 49, no. 11, Nov. 2001.

[6] Yuan-Pei Lin and See-May Phoong, "BER Minimized Transceivers with Channel Independent Transmitter," in preparation.

[7] N. Ahmed and K. R. Rao, *Orthogonal Transforms for Digital Signal Processing*, Springer Verlag, 1975.