STATISTICAL DESIGNS FOR TRANSMISSION OVER CORRELATED MIMO CHANNELS WITH LINEAR RECEIVERS

Yuan-Pei Lin, Chunwei Hsiao, and Chien-Chang Li

Dept. Electrical Engr., National Chiao Tung Univ., Hsinchu, Taiwan

ABSTRACT

In this paper we jointly consider statistical precoding and statistical bit allocation for transmission over correlated MIMO (multiple-input and multiple-output) channels with a linear and zero forcing receiver. Assuming the statistics of the channel is available to the transmitter, we will derive BER bounds using the statistics of subchannel error variances. The bounds are then used to design statistical precoder and statistical bit allocation. The combination of statistical precoding and bit allocation has great advantage over statistical precoding alone or statistical bit allocation alone. This is because the proposed precoding helps to bring out the statistical difference among the subchannels, which is then exploited by statistical bit allocation to better the performance. Simulations will be given to demonstrate that, the proposed system has significant gain over statistical designs that employs statistical precoding only or statistical bit allocation only.

1. INTRODUCTION

MIMO systems with limited feedback have received great interest recently. The system performance in terms of transmission rate or error rate can be improved significantly with limited amount of feedback from the receiver through a reverse channel. In particular precoded spatial multiplexing systems with finite-rate feedback have been investigated extensively. The receiver chooses the optimal precoder from a codebook and sends the index back to the transmitter. Optimal codebook designs of unitary precoders using Grassmannian subspace packing for different criteria are developed in [1]. The optimal unitary precoder for minimizing BER (bit error rate) using infinite feedback rate is given in [2] and generalized Lloyd algorithm is used for constructing codebooks. If, in addition to quantized precoder, power allocation and/or bit allocation is also available to the transmitter, the performance can be further improved [3]-[4]. Optimal designs of MIMO transceivers with bit loading have been developed in [5]. The use of identity precoder combined with feedback of only bit allocation is suggested therein as it intuitively requires less feedback. The feedback of only bit allocation for a fixed statistical precoder has been considered [6] for a correlated MIMO channel. In these works, instantaneous feedback is assumed to be available.

When only the channel statistics are available to the transmitter but not the current state of the channel, statistical precoder or statistical bit allocation has been proposed in the literature. The update of channel statistics requires only infrequent feedback and these statistical designs are very useful when there is no instantaneous feedback. For example, statistical precoders for linear receivers are considered in [7, 8, 9]. The precoder is designed to minimize an error rate bound in [7] and optimization of the precoder for minimum pairwise error probability is proposed in [9]. The optimal precoder that minimizes the sum of mean squared error for a receiver with decision feedback is given in [10]. In these works [7]-[10], a uniform bit allocation is assumed. Optimization of precoders for a fixed bit allocation has been developed in [11]. Statistical bit allocation is designed in [12] for uncorrelated channels based on capacity modeling. Statistical bit allocation for correlated channels is derived in [13] by combining antenna selection and bit loading. Statistical rate and power allocation are considered in [14] for the case the receiver has decision feedback, and asymptotic performance is analyzed. Statistical bit allocation and precoding have been considered in [15] for a decision feedback receiver.

In this paper we jointly consider statistical designs of precoding and bit allocation for transmission over correlated MIMO channels with a linear and zero-forcing receiver. Assuming the transmitter knows the statistics of the channel, we derive the precoding matrix and bit allocation to minimize BER bounds. We will see that with precoding, the channel is decorrelated and the disparity among the subchannels is made more pronounced. The resulting statistical difference of subchannels is then exploited using bit allocation. While systems with only precoding tend to allocate more power to subchannels with low SNR, systems with only bit allocation are not adequate to fully exploit the statistics, especially for more correlated channels. With the

This work was supported in parts by National Science Council, Taiwan, R. O. C., under NSC 98-2221-E-048-MY3 and International Research Center of Excellence on Advanced Bioengineering Grant NSC 99-2911-I-009-101.

978-1-4673-2569-1/12/$31.00 ©2012 IEEE

proposed combination of statistical precoding and bit allocation very good performance can be obtained compared to that with statistical precoding alone or with statistical bit allocation alone, as will be demonstrated in simulation examples.

The sections are organized as follows. In Sec. 2, we give the MIMO system model of the proposed system. In Sec. 3., statistical precoder and bit allocation are derived. Simulation results are given in Sec. 4 and a conclusion is given in Sec. 5. *Notation.* Boldfaced lower case letters represent vectors and boldfaced upper case letters are reserved for matrices. The notation $\mathbf{A}^{\dagger}$ denotes transpose-conjugate of $\mathbf{A}$. The function $E[y]$ denotes the expected value of a random variable y.

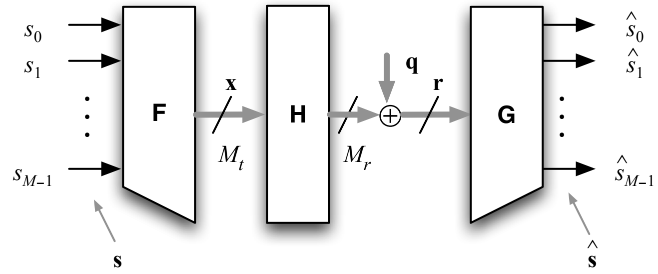

Figure 1: A MIMO system with M_t transmit antennas and M_r receive antennas.

2. SYSTEM MODEL OF THE STATISTICAL SYSTEM

Consider the MIMO system with M_t transmit antennas and M_r receive antennas in Figure 1. The channel is modeled by an $M_r \times M_t$ memoryless matrix $\mathbf{H}$ with $M_r \times 1$ additive white Gaussian channel noise $\mathbf{q}$ that has zero mean and variance N_0. The channel considered in this paper is of the form

$$\mathbf{H} = \mathbf{H}_w \mathbf{R}_t^{1/2}, \tag{1}$$

where $\mathbf{H}_w$ is an $M_r \times M_t$ matrix whose elements are independent Gaussian random variables with unit variance and $\mathbf{R}_t$, of dimensions $M_t \times M_t$, is the transmit correlation matrix. This model is useful for downlink transmission when the receive antennas are well separated. The precoder $\mathbf{F}$ is an $M_t \times M$ matrix, where $M \leq \min(M_t, M_r)$ is the number of subchannels. The $M \times 1$ input vector $\mathbf{s}$ consists of uncorrelated modulation symbols, $s_0, s_1, \cdots, s_{M-1}$, of zero mean and unit variance. The total transmission power $E[\mathbf{x}^{\dagger}\mathbf{x}]$ is P_t, where $\mathbf{x}$ is the transmitter output as indicated in Figure 1. The actual number of symbols transmitted can be smaller than M if one or more subchannels are loaded with zero bits. The receiver is linear and zero forcing. The $M \times M_r$ receive matrix $\mathbf{G}$ is given by $(\mathbf{F}^{\dagger}\mathbf{H}^{\dagger}\mathbf{H}\mathbf{F})^{-1}\mathbf{F}^{\dagger}\mathbf{H}^{\dagger}$ [16]. Define the error vector at the output of the receiver as $\mathbf{e} = \widehat{\mathbf{s}} - \mathbf{s}$. It has autocorrelation matrix $\mathbf{R}_e = E[\mathbf{e}\mathbf{e}^{\dagger}]$ given by [16]

$$\mathbf{R}_e = N_0(\mathbf{F}^{\dagger}\mathbf{H}^{\dagger}\mathbf{H}\mathbf{F})^{-1}. \tag{2}$$

The k-th subchannel error is $\sigma_{e_k}^2 = [\mathbf{R}_e]_{kk}$ and the error variance averaged over the random channel is $\overline{\sigma}_{e_k}^2 = E[\sigma_{e_k}^2]$.

It has been shown that $\prod_{k=0}^{M-1} \overline{\sigma}_{e_k}^2$ can be bounded using the eigen values of $\mathbf{R}_t$ [6]. Let the eigen decomposition of $\mathbf{R}_t$ be $\mathbf{U}_t \mathbf{\Lambda}_t \mathbf{U}_t^{\dagger}$, where $\mathbf{\Lambda}_t$ is a diagonal matrix with diagonal elements $\lambda_{t,i}$ ordered by $\lambda_{t,0} \geq \lambda_{t,1} \geq \cdots \lambda_{t,M_t-1}$. Assume $\lambda_{t,M-1} > 0$ and $M_r > M$, then $\prod_{k=0}^{M-1} \overline{\sigma}_{e_k}^2$ is bounded by

$$\prod_{k=0}^{M-1} \overline{\sigma}_{e_k}^2 \geq \prod_{k=0}^{M-1} \frac{N_0}{P_t/M}(M_r - M)^{-1}\lambda_{t,k}^{-1}. \tag{3}$$

The above inequality becomes an equality when we choose $\mathbf{F} = \sqrt{P_t/M}\mathbf{U}_{t,M}$, where $\mathbf{U}_{t,M}$ is the submatrix of $\mathbf{U}_t$ that consists of the first M columns of $\mathbf{U}_t$.

3. STATISTICAL BIT ALLOCATION AND STATISTICAL PRECODER

In this section, we will consider statistical designs of bit allocation and precoder. Assuming the inputs s_k are b_k-bit QAM symbols, the number of bits transmitted per channel use R_b is thus $\sum_{k=0}^{M-1} b_k$. The kth symbol error rate (SER) is well approximated by [17]:

$$SER_k = 4(1 - 2^{-b_k/2})Q\left(\sqrt{\frac{3}{(2^{b_k} - 1)\sigma_{e_k}^2}}\right), \tag{4}$$

where $Q(y) = 1/\sqrt{2\pi}\int_y^{\infty} e^{-t^2/2}dt, y \geq 0$. Assume the transmission rate is high and b_k is large enough so that $1 - 2^{-b_k/2} \approx 1$ and $2^{b_k} - 1 \approx 2^{b_k}$, then

$$SER_k \approx 4Q\left(\sqrt{3 \cdot 2^{-b_k}\sigma_{e_k}^{-2}}\right), \tag{5}$$

When Gray code is used, the kth subchannel BER can be approximated by $BER_k \approx SER_k/b_k$. Then the instantaneous BER averaged over the M subchannels $BER = \frac{1}{R_b}\sum_{k=0}^{M-1} b_k BER_k$ can be approximated by

$$BER \approx \frac{1}{R_b}\sum_{k=0}^{M-1} 4Q\left(\sqrt{3 \cdot 2^{-b_k}\sigma_{e_k}^{-2}}\right). \tag{6}$$

It is shown in [16] that $Q(\sqrt{3/y})$ is a convex function of y for $y \leq 1$ and a concave function of y for $y > 1$.

Consider first the case that the SNR is low so that $2^{b_k}\sigma^2_{e_k} > 1$ and the concavity of $Q(\sqrt{3/y})$ holds. Using Jensen's inequality for concave functions, we have

$$Q\left(\sqrt{3 \cdot 2^{-b_k}\sigma^{-2}_{e_k}}\right) \lesssim Q\left(\sqrt{3 \cdot 2^{-b_k}\overline{\sigma}^{-2}_{e_k}}\right), \quad (7)$$

where $\overline{\sigma}^2_{e_k} = E[\sigma^2_{e_k}]$ is the kth error variance averaged over the channel $\mathbf{H}$. Then the BER in (6) averaged over the channel, can be bounded as $E[BER] \lesssim \phi$, where

$$\phi = \frac{4}{R_b}\sum_{k=0}^{M-1} Q\left(\sqrt{\frac{3}{2^{b_k}\overline{\sigma}^2_{e_k}}}\right). \quad (8)$$

As $Q(\sqrt{3/y})$ is concave, we have

$$\phi \leq \frac{4M}{R_b} Q\left(\sqrt{\frac{3}{\frac{1}{M}\sum_{k=0}^{M-1} 2^{b_k}\overline{\sigma}^2_{e_k}}}\right). \quad (9)$$

Using $\sum_{k=0}^{M-1} b_k = R_b$ and the arithmetic mean-geometric mean inequality, the right hand side of (9) has the following lower bound $\phi_1 = \frac{4M}{R_b} Q\left(\sqrt{3 \cdot 2^{-R_b/M}\prod_{k=0}^{M-1}\overline{\sigma}^{-2/M}_{e_k}}\right)$ The lower bound is achieved if and only if $2^{b_k}\overline{\sigma}^2_{e_k}$ are of the same value for all k. This requires

$$b_k = \frac{1}{M}\sum_{\ell=0}^{M-1}\log_2(\overline{\sigma}^2_{e_\ell}) - \log_2(\overline{\sigma}^2_{e_k}) + R_b/M. \quad (10)$$

In this case, the inequality in (9) also becomes an equality and thus $\phi \approx \phi_1$, a quantity that is independent of bit allocation. Using the inequality in (3), we have $\phi \geq \phi_2$, where

$$\phi_2 = \frac{4M}{R_b} Q\left(\sqrt{3 \cdot 2^{-\frac{R_b}{M}}(M_r - M)\frac{P_t}{MN_0}\prod_{k=0}^{M-1}\lambda^{1/M}_{t,k}}\right), \quad (11)$$

and the equality holds when $\mathbf{F} = \sqrt{P_t/M}\mathbf{U}_{t,M}$ and bits allocated as in (10). The above precoder has also been shown to be optimal for different transmission systems in the literature, eg., [9, 15, 18].

Now consider the case SNR is large so that $2^{b_k}\sigma^2_{e_k} < 1$ and the convexity of $Q(\sqrt{3/y})$ holds. We can apply Jensen's inequality for convex functions, then the inequality in (7) is reversed and ϕ in (8) becomes a lower bound of BER. Therefore ϕ is an upper bound of BER in low SNR region and a lower bound in high SNR region. For low SNR we would like to have ϕ minimized. For high SNR we would also like to have ϕ minimized because the BER can not be small if ϕ is large. Using an approach similar to that in the low SNR case, it can be verified that ϕ is minimized for the same choices of bit allocation and precoder.

Note that when the precoder $\mathbf{F} = \sqrt{P_t/M}\mathbf{U}_{t,M}$, the channel is decorrelated as the equivalent channel seen by the input of the transmitter is

$$\mathbf{HF} = \mathbf{H}_w\mathbf{U}_t\mathbf{\Lambda}_t^{1/2}\begin{bmatrix}\mathbf{I}_M \\ \mathbf{0}\end{bmatrix}. \quad (12)$$

As $\mathbf{U}_t$ is unitary, $\mathbf{HF}$ is statistically the same as [19]

$$\mathbf{H}_w\mathbf{\Lambda}_t^{1/2}\begin{bmatrix}\mathbf{I}_M \\ \mathbf{0}\end{bmatrix}. \quad (13)$$

Effectively, the transmitter inputs are first scaled and then passed to the uncorrelated channel $\mathbf{H}_w$. In this case the kth subchannel error variance averaged over the channel is given by

$$\overline{\sigma}^2_{e_k} = \frac{N_0}{(M_r - M)P_t/M}\lambda^{-1}_{t,k}, \quad (14)$$

which is inversely proportional to the eigen value $\lambda_{t,k}$. For subchannels with larger $\lambda_{t,k}$ the corresponding subchannel error variances are statistically smaller. The statistical difference among the subchannels can then be exploited using bit allocation to have a lower error rate. The bit allocation formula in (10) in general yields real numbers. For high SNR, we can obtain the optimal integer bit allocation using the greedy algorithm in [20] because for high SNR ϕ is a convex function of b_k for a given precoder. So the greedy algorithm can be applied to find the optimal integer bit allocation. The bit allocation thus computed depends on the SNR P_t/N_0. However simulations show that the bit allocation is insensitive to SNR. We get the same optimal integer bit allocation for moderate to high SNR. Note that the BER bound in (11) depends on M. We can evaluate the bound for each M between 0 and M_t and choose the one that has the smallest BER bound.

4. SIMULATION EXAMPLE

The exponential correlation model in [21] is used to generate the random channel, in which the Hermitian matrix $\mathbf{R}_t$ is given by $[\mathbf{R}_t]_{mn} = \gamma^{n-m}$, for $0 \leq m \leq n < M_t$, where γ is the correlation coefficient between neighboring antennas. In the following Monte Carlo simulations, 10^6 channel realizations are used. The statistical bit allocation for the proposed system is computed using the greedy algorithm for high SNR P_t/N_0.

Example 1. In this example, $M_r = 6$, $M_t = 4$, $R_b = 8$, and $\rho = 0.7$. We plot the BER performance of the proposed system with statistical precoding and statistical bit allocation for different number of subchannels M in Figure 2. We have also shown the statistical bound ϕ_2 computed in (11) for different M. The curve of ϕ_2 for $M = 2$ falls below those for $M = 3$, $M = 4$ and $M = 1$. We can see that the BER for $M = 2$ is the smallest, followed by the cases

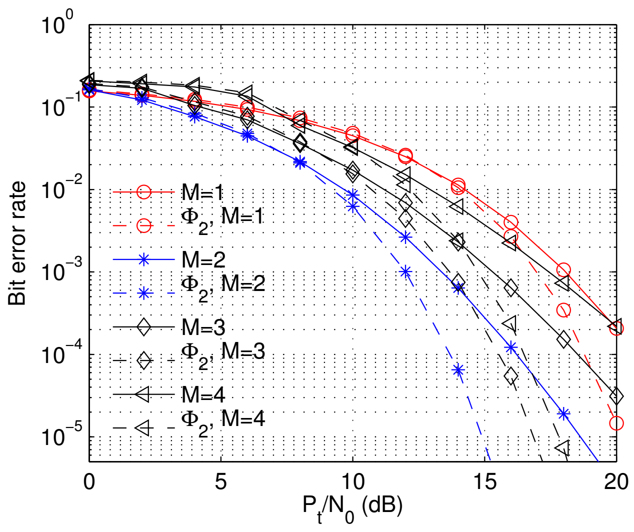

Figure 2: BER simulations for different number of subchannels M.

of $M = 3$, $M = 4$ and $M = 1$, in the same order as the curves of ϕ_2. Although minimizing ϕ_2 does not guarantee that the BER is also minimized, the statistical bound ϕ_2 is a good reference for determining the number of substreams to be transmitted. We will use ϕ_2 to determine M for the proposed system in the following example.

Example 2. In this example, $M_r = 5$, $M_t = 4$, and $R_b = 12$. The BER of the proposed statistical Precoding and Bit allocation system (P-BA) is shown in Figure 3 for correlated channels with $\gamma = 0, 0.3$, and 0.7. The statistical integer bit allocation for these 3 cases are, respectively, $(4, 4, 4, 0)$, $(5, 4, 3, 0)$, and $(7, 5, 0, 0)$. For a more correlated channel, there is more disparity among the subchannel error variances. The resulting bit allocation is more skewed and bit allocation is particularly useful.

Also shown in the figure is the system 'BA', which uses statistical bit allocation but no precoding (identity precoder). The bit allocation is computed in the same way as the proposed system. In addition we have shown in the figure two statistical precoding systems with linear and zero forcing receivers,'$\mathrm{P_{KS}}$' [7] and '$\mathrm{P_{LLJ}}$' [9]. In [7] the precoder is designed to minimize a BER bound while in [9] the objective is the pairwise symbol error probability and the optimal precoder is $\mathbf{U}_t$. Compared to these statistical designs with linear receivers, the proposed system requires a significantly smaller power for the same BER, especially when the correlation among the transmit antennas is high. For example, for BER $= 10^{-4}$, the gain of the proposed system over the two precoding systems is around 4 dB when $\rho = 0$ and the gain increases to 8.5 dB in the higher correlation case $\rho = 0.7$. The bit-allocation-only system 'BA' has the same performance as the 'P-BA' system when $\rho = 0$, but the gap widens as the correlation increases. The difference between 'P-BA' and '$\mathrm{P_{LLJ}}$' can be viewed as the gain of bit allocation as the precoders in both systems come from the eigen vectors of $\mathbf{R}_t$. Also the gap between 'P-BA' and 'BA' represents the gain of precoding as bit allocation in these two systems are computed in the same way. For comparison, we have also plotted the statistical precoding system with a decision feedback receiver, '$\mathrm{P_{LZW}}$' [10]. In [10] the design of the precoder involves more sophisticated optimization and the implementation of the receiver is more complex due to decision feedback. We observe that the proposed system has BER comparable to that of '$\mathrm{P_{LZW}}$' although 'P-BA' has a lower design and implementation cost.

5. CONCLUSION

In this paper, we proposed the combination of statistical precoding and statistical bit allocation for correlated MIMO channels with a linear and zero forcing receiver. We have derived statistical precoder and bit allocation for minimizing BER bounds when the receiver is linear and zero forcing. The statistical precoder decorrelates the channel and in the process brings out the statistical disparity among the subchannels. The disparity is then exploited using bit allocation. Simulations showed that a good performance can be achieved using the proposed statistical precoding and statistical bit allocation compared to systems that employ only statistical precoding or only statistical bit allocation.

6. REFERENCES

[1] D. J. Love, and R. W. Heath, Jr., "Limited feedback unitary precoding for spatial multiplexing systems," *IEEE Trans. Information Theory,* vol. 51, no. 8, Aug. 2005.

[2] S. Zhou, and B. Li, "BER criterion and codebook construction for finite-rate precoded spatial multiplexing with linear receivers," *IEEE Transaction on Signal Processing*, vol. 54, no. 5, pp. 1653-1665, May 2006.

[3] R. Yellapantula, Y. Yao, and R. Ansari,"Unitary precoding and power control in MIMO systems with limited feedback," *Proc. IEEE Wireless Communications and Networking Conference,* 2006.

[4] M. A. Sadrabadi, A. K. Khandani, and F. Lahouti, "Channel feedback quantization for high date rate MIMO systems," *IEEE Trans. Wireless Communications,* vol. 5, no. 12, Dec. 2006.

[5] C. C. Weng, C. Y. Chen, and P. P. Vaidyanathan "MIMO transceivers with decision feedback and bit loading: theory and optimization," *IEEE Trans. Signal Processing*, vol. 58, no. 3, March 2010.

[6] Y.-P. Lin, H.-C. Chen, and P. Jeng, "Bit allocation and statistical precoding for correlated MIMO channels with limited feedback," *IEEE Trans. Vehi. Tech.*, Feb. 2012.

[7] M. Kiessling and J. Speidel, "Statistical prefilter design for MIMO ZF and MMSE receivers based on majorization theory," *IEEE International Conf. Acoust., Speech, and Signal Processing*, 2004.

[8] S. H. Moon, J. S. Kim, and I. Lee, "Statistical precoder design for spatial multiplexing systems in correlated MIMO fading channels," *IEEE Vehicular Technology Conference*, 2010.

[9] Y. Li, H. Liu, and L. Jiang, "Statistical precoder design for spatial correlated MIMO channels in V-BLAST systems," *International Conference on Neural Networks and Signal Processing*, 2008.

[10] T. Liu, J.-K. Zhang, and K. M. Wong, "Optimal precoder design for correlated MIMO communication systems using zero-forcing decision feedback equalization," *IEEE Trans. Signal Processing*, vol. 57, no. 9, Sept. 2009.

[11] S. Järmyr, B. Ottersten, and E. A. Jorswieck "Statistical precoding with decision feedback equalization over a correlated MIMO channels," *IEEE Trans. Signal Processing*, vol. 58, no. 12, Dec. 2010.

[12] J. Du, Y. Li, D. Gu, A. F. Molisch, and J. Zhang, "Statistical Rate Allocation for Layered Space-Time Structure," *IEEE Trans. Communi.*, vol. 55, no. 3, March 2007.

[13] R. Narasimhan, "Spatial multiplexing with transmit antenna and constellation selection for correlated MIMO fading channels," *IEEE Trans. Signal Processing*, vol. 51, no. 11, Nov. 2003.

[14] H.-J. Lee, D.-M. Shin, K. Yang, "Asymptotic performance analysis of coded BlAST architectures with statistical rate and power allocation," *IEEE Trans. Communi.*, vol. 58, no. 8, Aug. 2010.

[15] Yuan-Pei Lin and See-May Phoong, "Statistical Bit Allocation and Statistical Precoding for Correlated MIMO Channels with Decision Feedback," to appear in IEEE Signal Processing Letters.

[16] P. P. Vaidyanathan, See-May Phoong, and Yuan-Pei Lin, "Signal processing and optimization for transceiver systems," Cambridge University Press, April 2010.

[17] J. G. Proakis, Digital communications, McGraw-Hill, 2001.

[18] S. A. Jafar and A. Goldsmith, "Transmitter optimization and optimality of beamforming for multiple antenna systems," *IEEE Trans. Wireless Communi.*, vol. 3, no. 4, July 2004.

[19] A. T. James, "Distributions of matrix variates and latent roots derived from normal samples," *The Annals of Mathematical Statistics,* vol. 35, no. 2, pp. 475-501, June, 1964.

[20] B. Fox, "Discrete optimization via marginal analysis," *Management Science*, vol. 13, no. 3, Nov. 1966.

[21] S. L. Loyka, "Channel capacity of MIMO architecture using the exponential correlation matrix," *IEEE Communi. Letters*, vol. 5, no. 9, pp. 369-371, Sept. 2001.

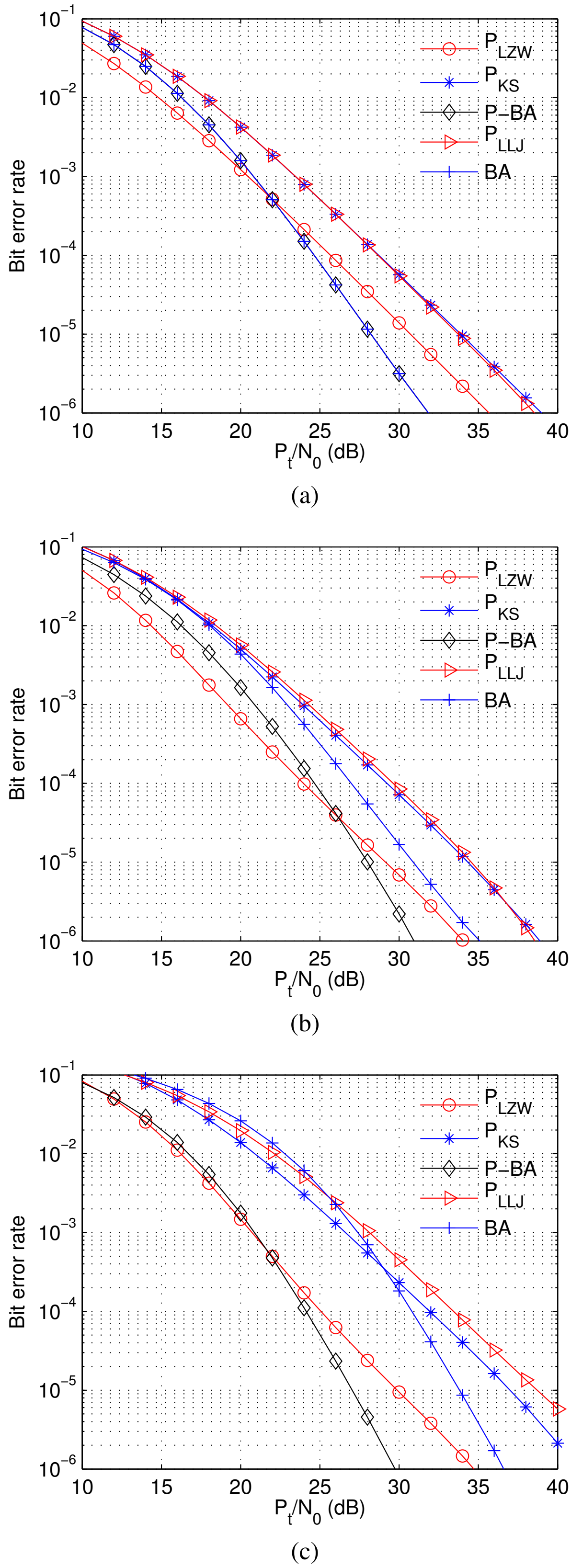

Figure 3: BER comparisons for different correlation coefficients, (a) $\gamma = 0$, (b) $\gamma = 0.3$, and (b) $\gamma = 0.7$.