FINDING THE MINIMUM SAMPLING FREQUENCY OF MULTI-BAND SIGNALS: AN EFFICIENT ITERATIVE ALGORITHM

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ABSTRACT

In this paper, we propose an efficient iterative algorithm for finding the minimum sampling frequency for a signal that consists of multiple bandpass signals. This finds important application in software radio where it is desirable to downconvert multiple bandpass signals simultaneously. We will derive a new set of conditions for alias-free sampling for signals that contain multi-band signals. The conditions can be easily examined with few computations. The minimum sampling frequency can be found by iteratively increasing the sampling frequency to meet the alias-free conditions. The simulations demonstrate that the proposed method requires a much lower complexity than existing algorithms.

1. INTRODUCTION

Bandpass sampling has important applications in downcoverting radio frequency (RF) signals. In the application of software defined radio systems, it is desirable to downconvert multiple RF signals simultaneously to save cost [1, 2]. The signal to be sampled may consist of more than one bandpass signal. Sampling theorem for a bandpass signal (two passbands) is well-known [3, 4]. The minimum frequency for alias-free sampling can be found in a closed form [5]. The minimum sampling frequency is usually significantly lower than the carrier frequency of the bandpass signal.

For signals with more than two passbands, the minimum sampling frequency can not be found in a closed from due to the nonlinear nature of spectrum folding in the process of sampling. An example of a spectrum that consists of N bandpass signals is shown in Fig. 1. Sampling for multi-band signals is extended in [2] and conditions for alias-free sampling derived. A systematic algorithm for finding valid sampling frequencies is developed in [6]. In [7][8][9], the complexity for finding valid sampling frequency is considerably reduced by imposing constraints on the ordering of the bands in the folded spectrum. These results may not yield the minimum frequency for alias-free sampling due to the ordering constraints. An efficient algorithm for finding valid sampling frequency range is proposed in [10]. By exhausting all possible orderings of the bands in the folded spectrum and categorizing all possible cases, the computational complexity can be reduced. An algorithm for finding the minimum sampling frequency is developed in [11] by finding the intersection of valid sampling frequencies for every two signal bands. An iterative algorithm for finding the minimum sampling frequency of signals that contains only two bandpass signals is given in [16].

In this paper, we propose an efficient algorithm for finding the minimum sampling frequency for a signal consisting of multiple bandpass signals. Generalizing the results in [16], we will derive a new set of conditions for alias-free sampling of multiband signals. These conditions can be checked with very few computations. When one of these conditions is not satisfied, the sampling frequency can be adjusted with minimum increment so that the condition becomes satisfied. By iteratively increasing the sampling frequency to meet the conditions for alias-free sampling, an algorithm for finding the minimum sampling frequency can be developed. There is no need to consider the ordering of the signal band in the folded spectrum. We will see that the algorithm based on the conditions derived in this paper is more efficient than previously reported methods.

The rest of the paper is organized as follows. We derive conditions for alias-free sampling of multiband signals in Sec. 2. Based on these conditions, an algorithm for finding the minimum sampling frequency of multiband signals is given in Sec. 3. Simulation examples are presented in Sec. 4 and a conclusion is given in Sec. 5.

2. CONDITIONS FOR ALIAS-FREE SAMPLING

Conditions for alias-free sampling can be stated in various ways in terms of the band edges and bandwidths of the member bandpass signals. The conditions that are employed affect the complexity of ensuing algorithms. In this section, we derive a new set of conditions for alias-free sampling that will lead to an efficient algorithm in the next section.

Suppose we are to sample a signal $X(f)$ that consists of multiple bandpass signals $X_1(f), X_2(f), \cdots, X_N(f)$ as shown in Fig. 1. Assume $X_i(f) \neq 0, \quad f_{\ell_i} < |f| < f_{h_i}$,

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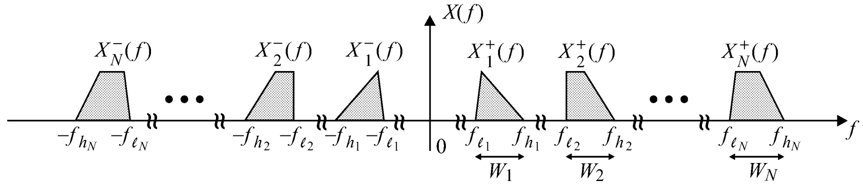

Figure 1: An example of spectrum that consists of N bandpass signals.

for $i = 1, 2, \cdots, N$, where f_{ℓ_i} and f_{h_i} are band edges, and $W_i = f_{h_i} - f_{\ell_i}$ are one-sided bandwidths as indicated in the figure. Let $X_i^+(f)$, and $X_i^-(f)$ denote respectively the positive frequency part and negative frequency part of $X_i(f)$. There are $2N$ signal bands, including $X_1^+(f), \cdots, X_N^+(f)$ and $X_1^-(f), \cdots, X_N^-(f)$. Since the replicas of any two bands may overlap and result in aliasing after sampling, there are a total of C_2^{2N} cases. Note that $X_i^+(f)$ and $X_i^-(f)$ are symmetric with respect to 0 for $i = 1, 2, \cdots, N$. If $X_i^+(f)$ and $X_j^+(f)$ are not aliasing after sampling, then $X_i^-(f)$ and $X_j^-(f)$ will not be aliasing by symmetry for $1 \leq i < j \leq N$. Similarly, if $X_i^-(f)$ and $X_j^+(f)$ are not aliasing after sampling, then $X_i^+(f)$ and $X_j^-(f)$ will not be aliasing for $1 \leq i < j \leq N$. Thus, we only need to consider the following N^2 cases:

(A0) Cases to be considered

$$\begin{array}{lll} \text{(a)} & \{X_i^+(f), X_i^-(f)\}, & \text{for } 1 \leq i \leq N, \\ \text{(b)} & \{X_i^+(f), X_j^+(f)\}, & \text{for } 1 \leq i < j \leq N, \\ \text{(c)} & \{X_i^-(f), X_j^+(f)\}, & \text{for } 1 \leq i < j \leq N. \end{array} \quad (1)$$

To discuss the above different cases in a more general setting, we first consider the sampling of a hypothetical 2-band signal $Y(f)$ as shown in Fig. 2(a). $Y(f)$ consists of $P(f)$ and $Q(f)$, where $P(f) \neq 0$, only for $f_{p_1} < f < f_{p_2}$ and $Q(f) \neq 0$, only for $f_{q_1} < f < f_{q_2}$. The bandedges f_{p_1}, f_{p_2}, f_{q_1}, and f_{q_2} can be positive or negative.

Lemma 1 *For the 2-band signal $Y(f)$ in Fig. 2(a), there is no aliasing for a given sampling frequency f_s if and only if*

$$\begin{array}{rl} & (f_{q_2} - f_{p_1}) \pmod{f_s} = 0, \\ or & (f_{q_2} - f_{p_1}) \pmod{f_s} \geq W_p + W_q, \end{array} \quad (2)$$

where $W_p = f_{p_2} - f_{p_1}$, and $W_q = f_{q_2} - f_{q_1}$.

Proof. We observe that there is no aliasing in sampling $Y(f)$ if and only if there is no aliasing when we sample a shifted version $Y(f + f_0)$, where f_0 is the shift. For convenience we will consider the condition for alias-free sampling of $Y(f + f_0)$. Suppose we choose f_0 as the midpoint of f_{p_1} and f_{q_2}, i.e.,

$$f_0 = (f_{p_1} + f_{q_2})/2.$$

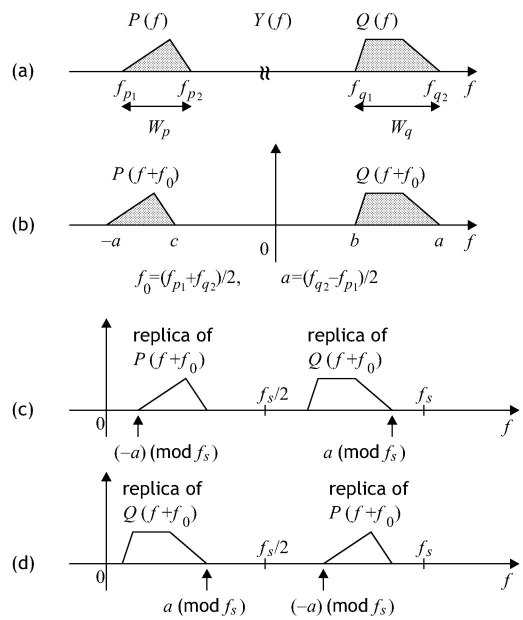

Figure 2: (a) The spectrum of a hypothetical 2-band signal $Y(f)$. (b) $Y(f + f_0)$, where $f_0 = (f_{p_1} + f_{q_2})/2$. (c) An example of the folded spectrum for the interval $[0, f_s)$ when $a \pmod{f_s} \geq (-a) \pmod{f_s}$. (d) An example of the folded spectrum for the interval $[0, f_s)$ when $a \pmod{f_s} < (-a) \pmod{f_s}$.

Then the shifted pair is as shown in Fig. 2(b), where $a = (f_{q_2} - f_{p_1})/2$, $b = f_{q_1} - (f_{p_1} + f_{q_2})/2$, $c = f_{p_2} - (f_{p_1} + f_{q_2})/2$. If we consider the folded spectrum in the $[0, f_s)$ interval, the band edges $a \pmod{f_s}$ and $(-a) \pmod{f_s}$ are equal-distanced from $f_s/2$. We now discuss two possible scenarios: (i) $a \pmod{f_s} \geq (-a) \pmod{f_s}$ and (ii) $a \pmod{f_s} < (-a) \pmod{f_s}$. Examples of these two possible cases are shown respectively in Fig. 2(c) and (d).

(i) When $a \pmod{f_s} \geq (-a) \pmod{f_s}$ there will be no aliasing if and only if $(-a) \pmod{f_s} = a \pmod{f_s}$ or if the interval $((-a) \pmod{f_s}, a \pmod{f_s})$ is large enough to accommodate the two replicas. That is, $x = 0$, or $x \geq W_p + W_q$, where $x = a \pmod{f_s} - ((-a) \pmod{f_s})$. The equivalent conditions are

$$2a \pmod{f_s} = 0, \text{ or } 2a \pmod{f_s} \geq W_p + W_q. \quad (3)$$

(ii) When $a \pmod{f_s} < (-a) \pmod{f_s}$ as shown in Fig. 2(d), there is some space between the two replicas and the space is of length $((-a) \pmod{f_s} - a \pmod{f_s})$. There will be no aliasing if and only if the remaining part of the $[0, f_s)$ interval is large enough to take in the two replicas. That is, $f_s - ((-a) \pmod{f_s} - a \pmod{f_s}) \geq W_p + W_q$, Or equivalently $2a \pmod{f_s} \geq W_p + W_q$. This is the same as the second condition in (3).

Substituting $a = (f_{q_2} - f_{p_1})/2$ to (3), we obtain the necessary and sufficient condition for alias-free sampling of $Y(f)$ in (2). ■

The result in Lemma 1 is for the sampling of a 2-band signal with arbitrary band locations. We can apply it to each of the cases in (A0). Then we can obtain sufficient and necessary conditions for aliasfree sampling of multiband signals. For a given sampling frequency f_s, there will not be aliasing if and only if the following are true.

(A1) Aliasfree conditions

(a) $2f_{h_i} \pmod{f_s} = 0$, or $2f_{h_i} \pmod{f_s} \geq 2W_i$, for $1 \leq i \leq N$.

(b) $(f_{h_j} - f_{\ell_i}) \pmod{f_s} = 0$, or $(f_{h_j} - f_{\ell_i}) \pmod{f_s} \geq W_i + W_j$, for $1 \leq i < j \leq N$.

(c) $(f_{h_i} + f_{h_j}) \pmod{f_s} = 0$, or $(f_{h_i} + f_{h_j}) \pmod{f_s} \geq W_i + W_j$, for $1 \leq i < j \leq N$.

3. AN ALGORITHM FOR FINDING THE MINIMUM SAMPLING FREQUENCY

For a given sampling frequency f_s, there will be no aliasing if all the conditions in (A1) are met. If any one of the conditions is not satisfied, we will see how to make minimum increment to the sampling frequency so that the condition becomes satisfied.

Let us first go back to the hypothetical 2-band signal $Y(f)$ that is useful in previous section.

Lemma 2 *Consider the sampling of the 2-band signal $Y(f)$ in Fig. 2. Suppose there is aliasing for a given sampling frequency f_s. Then the smallest $f_{s,new} > f_s$ that yields aliasfree sampling of $Y(f)$ is*

$$f_{s,new} = \frac{f_{q_2} - f_{p_1}}{\lfloor (f_{q_2} - f_{p_1})/f_s \rfloor}. \quad (4)$$

Proof. Consider the folded spectrum in the interval $[0, f_s)$ as shown in Fig. 2(c) and (d). We discuss the two cases: (i) $a \pmod{f_s} < f_s/2$ and (ii) $f_s/2 < a \pmod{f_s} < f_s$, separately. (i) $0 < a \pmod{f_s} < f_s/2$: When we gradually increase the sampling frequency the band edge $a \pmod{f_s}$ of replica $Q(f)$ moves towards 0 while the band edge $(-a) \pmod{f_s}$ of replica $P(f)$ moves towards f_s. When the sampling frequency is increased such that $a \pmod{f_s}$ decreases to 0, then the condition in (2) becomes satisfied. (ii) $f_s/2 < a \pmod{f_s} < f_s$: Similarly the condition in (2) becomes satisfied when $a \pmod{f_s}$ decreases to $f_s/2$.

Therefore we can conclude that the alias-free condition in (2) can be satisfied by increasing the sampling frequency such that a becomes an integer multiple of $f_s/2$. The smallest new sampling $f_{s,new}$ for this to happen can be computed as follows. Let us write a as $a = n_a f_s/2 + r_a$, where $r_a = a \pmod{f_s/2}$ and $n_a = \lfloor a/(f_s/2) \rfloor$. Then we have $a = n_a f_{s,new}/2$, or $f_{s,new} = 2a/n_a$. Using the fact that n_a can also be computed using $n_a = \lfloor 2a/f_s \rfloor$, we obtain the expression of $f_{s,new}$ in (4). ■

We can apply Lemma 2 to the cases in (A0). Then for each case in (A0), we can obtain a formula for adjusting the sampling frequency so that the corresponding aliasfree condition in (A1) is satisfied.

(A2) Frequency adjustment formula

(a) $f_{s,new} = \frac{2f_{h_i}}{\lfloor (2f_{h_i})/f_s \rfloor}$, for $1 \leq i \leq N$.

(b) $f_{s,new} = \frac{f_{h_j} - f_{\ell_i}}{\lfloor (f_{h_j} - f_{\ell_i})/f_s \rfloor}$, for $1 \leq i < j \leq N$.

(c) $f_{s,new} = \frac{f_{h_i} + f_{h_j}}{\lfloor (f_{h_i} + f_{h_j})/f_s \rfloor}$, for $1 \leq i < j \leq N$.

Proposed iterative algorithm. Using the conditions for alias-free sampling in section 2 and the methods for computing new sampling frequency for each case, we have the following iterative algorithm for finding the minimum sampling frequency. To start off, let $f_s = 2(W_1 + W_2 + \cdots + W_N)$, which is the lowest possible sampling frequency for no aliasing.

1. Examine the conditions for aliasfree sampling in (A1) one by one. If any one of the condition is not satisfied, go to the next step. If all the conditions in (A1) are satisfied, then we have found the minimum sampling frequency.
2. For the condition that is violated in Step 1, compute the corresponding new sampling frequency using (A2). Go to Step 1.

There is no need considering the ordering of signal bands in the folded spectrum. The conditions in (A1) can be easily examined and frequency adjustment in (A2) can be done with few computations. As a result, the proposed method requires a lower complexity than earlier methods as will be demonstrated in the next section.

4. SIMULATIONS AND COMPARISONS

In this section, we apply the proposed algorithm to wireless applications. The bandpass signals considered in the simulations are GSM 900 (935-960 MHz, one-sided bandwidth

Case	Method in [10]		Method in [11]		Proposed Method	
	ADD	MUL	ADD	MUL	ADD	MUL
GSM900, GSM1800, 802.11g	105	186	87	109	60	42
DAB, GSM1800, 802.11g	75	126	99	133	41	36
GSM900, DAB, WCDMA	183	342	198	331	77	84

Table 1: Complexity for finding the minimum sampling frequency of multiple bandpass signals in terms of additions (ADD) and multiplications (MUL).

25 MHz), GSM 1800 (1805-1880 MHz, one-sided bandwidth 75 MHz) [13], DAB Eureka-147 L-Band (1472.286-1473.822 MHz, one-sided bandwidth 1536 KHz) [14], IEEE 802.11g (2412-2432 MHz, one-sided bandwidth 20 MHz) [15], and WCDMA (2119-2124 MHz, one-sided bandwidth 5 MHz). Table 1 lists the complexity of finding the minimum sampling frequency for different combinations of bandpass signals and compares with the methods in [10][11]. The complexity is given in terms of numbers of multiplications (MUL) and additions (ADD). The simulation result demonstrates that the proposed method can reduce the number of additions and multiplications significantly. The required numbers of additions and multiplications are reduced respectively by around 28-58% and 61-76%.

5. CONCLUSIONS

We have proposed an efficient algorithm for finding the minimum sampling frequency for signals that contain multipassband signals. We have derived a new set of necessary and sufficient conditions for alias-free sampling that can be checked with few computations. There is no need to consider ordering of the signal bands in the folded spectrum. The conditions developed in this paper lead to an efficient iterative algorithm for finding the minimum sampling frequency. The complexity is much lower than existing methods.

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