AN IMPROVED DESIGN OF DFT BANK TRANSCEIVERS FOR UNKNOWN CHANNELS

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ABSTRACT

In recent years, there has been considerable interest in applying filter bank (FB) technique for transceiver design. Though their filters have good frequency responses, FB transceivers often suffer from severe intersymbol interference (ISI) when the channel is frequency selective. Additional post processing is needed at the receiver. Recently a new technique in designing DFT bank transceivers for unknown channels was proposed. Though the transceiver is DFT modulated, the only channel dependent part is a set of scalars at the receiver. Like the OFDM system, simple equalization can be employed at the receiver. In this paper, we propose an improved method for designing such DFT bank transceivers. Simulation results show that DFT bank transceivers with better stopband attenuation and higher signal to interference ratio (SIR) can be achieved.

1. INTRODUCTION

The orthogonal frequency division multiplexing (OFDM) and discrete multitone modulation (DMT) systems have been widely adopted for broadband communications. These systems are DFT based and have low complexity. However their transmit and receive prototype filters are the rectangular windows and therefore have poor frequency responses. FB technique has been proposed for designing transceivers with good filters[1]-[4]. In these designs, perfect reconstruction FBs are often employed. However for frequency selective channels, these transceivers in general suffer from severe ISI and additional post processing is needed at the receiver [1,2,3].

Recently a new method for designing channel-resilient DFT bank transceiver is proposed [5]. The only channel dependent part of the DFT bank transceiver is a set of scalars and hence simple one-tap equalizers can be employed at the receiver for symbol recovery. Transceivers with high SIR can be obtained. However, only the transmit or receive filters (not both) are guaranteed to have good frequency responses. In many applications, we would like to have transmit and receive filters with a large stopband attenuation so that the out of band spectral leakage is low at the transmitter and narrowband interference is effectively filtered at the receiver. To achieve this goal, the authors in [6] developed an iterative procedure to design DFT bank transceivers with good responses as well as high SIR. Frequency constraints are imposed by splitting the prototype filters into two parts: a fixed prefilter for getting good response and a subfilter for maximizing SIR. Though satisfactory results can be obtained, the prefilter approach has some drawbacks.

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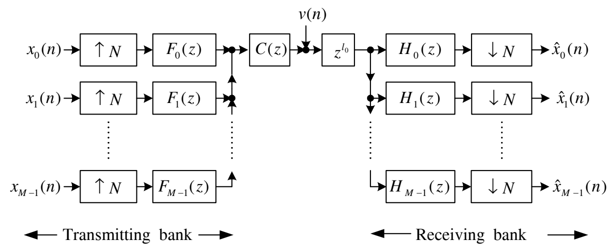

Figure 1: A filter bank transceiver.

Firstly the number of free parameters for frequency response and that for SIR maximization have to be adjusted by trial and error so that a good compromise between the frequency response and SIR is obtained. Secondly, though the prefilter has good response, the cascade of the prefilter and the SIR maximized subfilter may not be a good filter.

In this paper, we propose an improved method for designing DFT bank transceivers. The new method enables us to use all free parameters to optimize the frequency response and SIR simultaneously. Numerical simulation shows that the proposed method not only avoids the drawbacks of the prefilter approach, it also gives a better result.

2. DFT BANK TRANSCEIVERS

Fig. 1 shows a FB transceiver with M subbands. The upsampling ratio is N. We assume that $N \geq M$ and $N-M$ represents the number of redundant samples per block. We assume that input signals $x_i(n)$ are zero-mean random processes that satisfy

$$E[x_i(n)x_j^*(m)] = \mathcal{E}_x \delta(i-j)\delta(n-m). \tag{1}$$

The transmission channel is assumed to be an L-th order random channel $C(z) = \sum_{l=0}^{L} c(n)z^{-l}$. The channel taps $c(l)$ are complex random variables that satisfy

$$E[c(l)] = 0 \qquad E[c(l)c^*(l-k)] = \sigma_l^2 \delta(k). \tag{2}$$

An advance operator z^{l_0} is added at the receiver to adjust for the delay caused by $C(z)$. For most cases, l_0 can be chosen as $L/2$. The cost of FB transceiver is very high. We need to implement M filters at the transmitter (or receiver). To reduce the cost, modulated filters are often employed [1,3,4,5]. This paper considers DFT bank transceiver where the filters are related by:

$$F_i(z) = F_0(ze^{-j\frac{2\pi i}{M}}), \qquad H_i(z) = H_0(ze^{-j\frac{2\pi i}{M}})$$

for $0 \leq i \leq M-1$. The transmit and receive prototype filters are respectively given by

$$F_0(z) = \sum_{n=0}^{N_f} f_0(n) z^{-n}, \qquad H_0(z) = \sum_{n=0}^{N_h} h_0(n) z^{n}.$$

The filter orders N_f and N_h can be larger than N. We only need to implement a prototype filter and an IFFT (or FFT) at the transmitter (or receiver). When $N_f = N$ and $N_h = N$ and $f_0(n) = h_0(n) = 1$, the transceiver reduces to an OFDM or DMT system. Thus a DFT bank transceiver can be viewed as a generalization of OFDM and DMT systems.

Using multirate identities, the transfer function from the ith input to the jth output is LTI:

$$T_{j,i}(z) = \sum_{l=0}^{L} c(l) \left[F_i(z) H_j(z) z^{l_0 - l} \right]_{\downarrow N},$$

The notation $[\bullet]_{\downarrow N}$ denotes N-fold downsampling. Define

$$\left[F_i(z) H_j(z) z^{l_0 - l} \right]_{\downarrow N} = \sum_n \beta_{i,j,l}(n) z^{-n}, \tag{3}$$

for $0 \leq i, j \leq M-1$ and $0 \leq l \leq L$. Then we can express the output signal at the jth subband in the time domain as

$$\begin{aligned} \hat{x}_j(n) = & \left[\sum_{l=0}^{L} \beta_{j,j,l}(0) c(l) \right] x_j(n) \\ & + \sum_{l=0}^{L} c(l) \left(\beta_{j,j,l}(n) - \beta_{j,j,l}(0) \delta(n) \right) * x_j(n) \\ & + \sum_{i \neq j, i=0}^{M-1} \sum_{l=0}^{L} c(l) \beta_{i,j,l}(n) * x_i(n), \end{aligned}$$

where $*$ stands for convolution. The first, second, and the last terms of the above expression are the desired signal, the intra-band ISI and the cross-band ISI, respectively. Under the assumptions in (1) and (2), we can calculate the average signal and isi powers by taking the statistical expectation with respect to input signals $x_i(n)$ and channel taps $c(l)$. It was shown [5] that all the outputs have the same signal power and the same ISI power and they are given by

$$P_{sig} = \mathcal{E}_x \sum_{l=0}^{L} |\beta_{0,0,l}(0)|^2 \sigma_l^2$$

$$P_{isi} = \mathcal{E}_x \sum_{l=0}^{L} \left(\sum_{n \neq 0} |\beta_{0,0,l}(n)|^2 + \sum_{i \neq 0, n} |\beta_{i,0,l}(n)|^2 \right) \sigma_l^2.$$

Recall from (3) that $\beta_{i,0,l}(n)$ are the coefficients of the convolution of $f_i(n)$ and $h_0(n)$. Thus $\beta_{i,0,l}(n)$ can be written as a linear combination of $f_0(n)$ or $h_0(n)$. Define the vectors

$$\begin{aligned} \mathbf{f} &= \left[\begin{array}{cccc} f_0(0) & f_0(1) & \ldots & f_0(N_f) \end{array} \right]^T \\ \mathbf{h} &= \left[\begin{array}{cccc} h_0(0) & h_0(1) & \ldots & h_0(N_h) \end{array} \right]^T \\ \boldsymbol{\beta}_i(n) &= \left[\begin{array}{cccc} \beta_{i,0,0}(n) & \beta_{i,0,1}(n) & \ldots & \beta_{i,0,l}(n) \end{array} \right]^T. \end{aligned}$$

Then we can find matrices $\mathbf{A}_i(n)$ and $\mathbf{B}_i(n)$ such that

$$\boldsymbol{\beta}_i(n) = \mathbf{A}_i(n) \mathbf{f} = \mathbf{B}_i(n) \mathbf{h}. \tag{4}$$

The entries of $\mathbf{A}_i(n)$ and $\mathbf{B}_i(n)$ comprise of prototype coefficients $h_0(n)$ and $f_0(n)$ respectively. Using this relation, we can write

$$P_{sig} = \mathcal{E}_x \mathbf{f}^\dagger \underbrace{\mathbf{A}_0^\dagger(0) \mathbf{D} \mathbf{A}_0(0)}_{\mathbf{Q}_0} \mathbf{f}$$

$$P_{isi} = \mathcal{E}_x \mathbf{f}^\dagger \underbrace{\left[\sum_{i,n} \mathbf{A}_i(n) - \mathbf{A}_0(0) \right]^\dagger \mathbf{D} \left[\sum_{i,n} \mathbf{A}_i(n) - \mathbf{A}_0(0) \right]}_{\mathbf{Q}_1} \mathbf{f},$$

where the diagonal matrix $\mathbf{D} = diag[\sigma_0^2 \, \sigma_1^2 \, \ldots \, \sigma_L^2]$ and $\mathbf{f}\dagger$ is the transpose conjugate of $\mathbf{f}$. Hence the SIR can be written as

$$SIR = \frac{\mathbf{f}^\dagger \mathbf{Q}_0 \mathbf{f}}{\mathbf{f}^\dagger \mathbf{Q}_1 \mathbf{f}}. \tag{5}$$

Similarly, using (4) one can express the SIR in terms of $\mathbf{h}$ as

$$SIR = \frac{\mathbf{h}^\dagger \widetilde{\mathbf{Q}}_0 \mathbf{h}}{\mathbf{h}^\dagger \widetilde{\mathbf{Q}}_1 \mathbf{h}}, \tag{6}$$

for some matrices $\widetilde{\mathbf{Q}}_i$. Note that the matrices $\mathbf{Q}_i$ and $\widetilde{\mathbf{Q}}_i$ are positive semi-definite. The SIR can be expressed as a Rayleigh-Ritz ratio of $\mathbf{f}$ or $\mathbf{h}$. Given a fixed receive prototype filter $\mathbf{h}$, we can solve (5) for the best transmit prototype filter $\mathbf{f}$ so that the SIR is maximized, and vice versa. One can iteratively optimize the prototype filters $\mathbf{f}$ and $\mathbf{h}$ using (5) and (6) respectively and the SIR will increase monotonically. However the filters obtained by such an iterative procedure will eventually converge to the rectangular windows, which do not have good frequency responses. To get good filters, a prefilter method was proposed [6]. In this method, the transmit and receive prototype filters are split into two parts

$$H_0(z) = H_a(z) H_b(z), \qquad F_0(z) = F_a(z) F_b(z). \tag{7}$$

The prefilters $H_a(z)$ and $F_a(z)$ are fixed and designed as good lowpass filters whereas the subfilters $F_b(z)$ and $H_b(z)$ are iteratively optimized for SIR maximization. In the time domain, (7) can be written as:

$$\mathbf{h} = \mathbf{H}_a \mathbf{h}_b, \qquad \mathbf{f} = \mathbf{F}_a \mathbf{f}_b,$$

where $\mathbf{H}_a$ and $\mathbf{F}_a$ are the convolution matrix and $\mathbf{h}_b$ and $\mathbf{f}_b$ are vectors formed by the coefficients of $H_b(z)$ and $F_b(z)$ respectively. Substituting this relation into the expressions of SIR in (5) and (6), we have

$$SIR = \frac{\mathbf{f}_a^\dagger \mathbf{P}_0 \mathbf{f}_a}{\mathbf{f}_a^\dagger \mathbf{P}_1 \mathbf{f}_a} = \frac{\mathbf{h}_a^\dagger \widetilde{\mathbf{P}}_0 \mathbf{h}_a}{\mathbf{h}_a^\dagger \widetilde{\mathbf{P}}_1 \mathbf{h}_a}$$

where $\mathbf{P}_i = \mathbf{F}_a^\dagger \mathbf{Q}_i \mathbf{F}_a$ and $\widetilde{\mathbf{P}}_i = \mathbf{H}_a^\dagger \widetilde{\mathbf{Q}}_i \mathbf{H}_a$. The optimal subfilters $\mathbf{h}_b$ and $\mathbf{f}_b$ that maximize the SIR can be obtained by solving the corresponding Rayleigh-Ritz ratio.

3. THE PROPOSED METHOD

In the prefilter approach, the number of free parameters for frequency response as well as the number of free parameters for SIR maximization are fixed. Moreover we impose no frequency constraint on the subfilters $F_b(z)$ and $H_b(z)$. Because the prototype filters are the products $H_a(z)H_b(z)$ and $F_a(z)F_b(z)$, their frequency responses will be affected by the subfilters. In the following, we propose a new objective function that incorporates both the SIR and the frequency constraints. The new objective function enables us to exploit all degrees of freedom for simultaneous optimization of SIR and frequency responses. To do this, we consider the weighted stopband energy of the prototype filters:

$$E_f = \int_{\Re_f} W_f(\omega)|F_0(e^{j\omega})|^2 \frac{d\omega}{2\pi},$$
$$E_h = \int_{\Re_h} W_h(\omega)|H_0(e^{j\omega})|^2 \frac{d\omega}{2\pi}.$$

where $\Re$ is the stopband region and $W(\omega)$ is a nonnegative weighting function. It is well known that these stopband energies can be expressed in terms of the filter coefficients as

$$E_f = \mathbf{f}^\dagger \mathbf{Q}_2 \mathbf{f}, \;\; E_h = \mathbf{h}^\dagger \widetilde{\mathbf{Q}}_2 \mathbf{h},$$

for some positive definite matrices $\mathbf{Q}_2$ and $\widetilde{\mathbf{Q}}_2$. To incorporate these energies into the optimization, we define two new objective functions:

$$J_{Tx} = \frac{\mathbf{f}^\dagger \mathbf{Q}_0 \mathbf{f}}{\mathbf{f}^\dagger \left(\mathbf{Q}_1 + \alpha \mathbf{Q}_2\right) \mathbf{f}}$$
$$J_{Rx} = \frac{\mathbf{h}^\dagger \widetilde{\mathbf{Q}}_0 \mathbf{h}}{\mathbf{h}^\dagger (\widetilde{\mathbf{Q}}_1 + \widetilde{\alpha} \widetilde{\mathbf{Q}}_2) \mathbf{h}}$$

where α and $\widetilde{\alpha}$ are scaling factors controlling the relative weight between the ISI power and the stopband energy. These functions are also in the form of Rayleigh-Ritz ratios and the optimal filters are the eigenvectors of some associated matrices. Note that we do not include the passband responses in the objective functions. This is because of two reasons. Firstly the number of bands M is usually large in transceiver design and hence the filters have narrow passbands. Secondly, the passband responses are less important because the ability of signal recovery is guaranteed by the SIR value. Using the two new objective functions, we can alternatively optimize the two prototype filters by the following iterative procedure:

1. The transmit prototype filter $H_0^{(0)}(z)$ is initialized as a good lowpass filter. Then let $k = 1$.

For $k > 0$

2. Given transmit prototype filter $H_0^{(k-1)}(z)$, find $F_0^{(k)}(z)$ so that J_{Tx} is maximized.
3. Given receive prototype filter $F_0^{(k)}(z)$, find $H_0^{(k)}(z)$ so that J_{Rx} is maximized
4. Terminate if the improvement between the kth and $(k-1)$th iteration is less than the desired value or if it reaches the maximum number of iterations; otherwise, $k = k + 1$ and return to step 2.

4. SIMULATIONS

In this section, we carry out some simulations to verify the results. The number of subbands is $M = 64$ and the interpolation ratio is $N = 80$. The orders of the prototype filters $N_h = N_f = 160$. The random channels are 5-tap channels that satisfy the channel model given in (2). The taps are independently drawn from a circularly-symmetric complex Gaussian distribution with the same variance. Note that this is the widely adopted channel model in transceiver design when we do not have any channel information. The two prototype filters have real coefficients. The stopband regions of these filters are $\Re_h = \Re_f = [0.033\pi \;\; \pi]$. In the optimization, we set $\alpha = \widetilde{\alpha} = 3.25$. The weighting functions are

$$W_f(\omega) = W_h(\omega) = \begin{cases} |\omega|^4, & \omega \in [0.033\pi, 0.4\pi]; \\ (0.4\pi)^4, & \omega \in [0.4\pi, \pi]. \end{cases}$$

That is, we want higher stopband attenuation at higher frequency. For comparison, we also design DFT bank transceivers using the prefilter method with the same orders $N_h = N_f = 160$. The orders of the prefilters are $N_{ha} = N_{fa} = 60$ (this value is found experimentally and it gives a good tradeoff between the filter response and SIR).

The plot of SIR versus the number of iterations is shown in Fig. 2. We see that SIR increases with the number of iterations in both methods. The transceiver designed by the proposed method enjoys a higher SIR and the gain can be as large as 4 dB. We plot the magnitude responses of the prototype filters designed by both methods after 200 iterations. The plots are shown in Fig. 3 and 4 respectively. For the purpose of comparison, in these figures we also include the frequency responses of the rectangular windows which are the prototype filters in OFDM systems. We see that the frequency responses of both DFT bank transceivers are much better than the OFDM system. Moreover filters designed by the proposed method have a better stopband attenuation than the prefilter approach, especially at the receiver. Define the out of band energy E_o, the attenuation of the first sidelobe P_1, and the attenuation in the high frequency region $[0.5\pi \;\; \pi]$, P_2, respectively as

$$E_o = \frac{1}{\pi} \int_{2\pi/M}^{\pi} |T(e^{j\omega})|^2 d\omega$$
$$P_1 = - \max_{\omega \in [2\pi/M, \pi/2]} 20 \log_{10} |T(e^{j\omega})| (\text{dB})$$
$$P_2 = - \max_{\omega \in [\pi/2, \pi]} 20 \log_{10} |T(e^{j\omega})| (\text{dB}),$$

where $T(z) = F_0(z)$ at the transmitter and it is equal to $H_0(z)$ at the receiver. We calculate these quantities for the three transceivers and the results are listed in Table 1. It is clear that the proposed method has the best performance in almost every aspect.

In addition, we compare the bit rate performance of the three transceivers. A total of 100 sets of random channels are generated. The channel noise is complex AWGN with variance N_0. Both the cases of channels with or without narrowband interference (NBI) are considered. The narrowband noise is modeled as $\sqrt{30N_0}\sin(0.2\pi n)$. The number of bits per block, B, is calculated by $B = \sum_{i=0}^{M-1} \log_2 \left(1 + \gamma_i/\Gamma\right)$, where γ_i is the signal to interference plus noise ratio of the ith subband. We set $\Gamma = 10$ which corresponds to a bit error probability of around 10^{-7} for uncoded data. We plot B as a function of E_x/N_0, where E_x is the symbol energy. The results are shown in Fig. 5. When there is no NBI, the OFDM system performs slightly better than both

Table 1: Performance comparison.

	Tx			Rx		
	E_o	P_1(dB)	P_2(dB)	E_o	P_1(dB)	P_2(dB)
Propose	5.075 e-03	14.994	78.931	5.748 e-03	14.033	92.694
Prefilter	6.103 e-03	14.481	74.922	5.830 e-03	14.739	73.225
OFDM	1.130 e-02	13.282	35.125	1.517 e-02	13.256	33.321

the DFT bank transceivers. However, in the presence of NBI, both DFT bank transceivers are significantly better and the proposed method slightly outperforms the prefilter method.

5. CONCLUSIONS

In this paper, we propose a new method for designing DFT bank transceivers for unknown channels. Transceivers with good stopband attenuation and high SIR can be achieved. Experiment shows that the proposed method has a better performance comparing with the prefilter method.

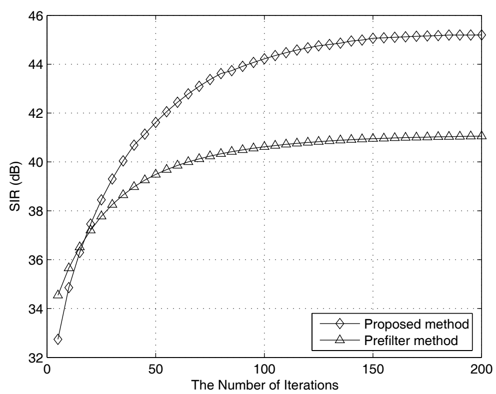

Figure 2: SIR vs the number of iterations.

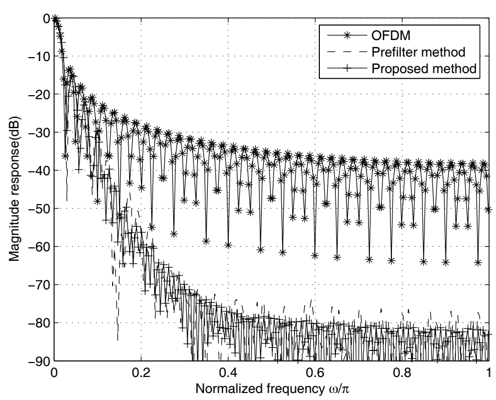

Figure 3: Magnitude response of the transmit prototype filters.

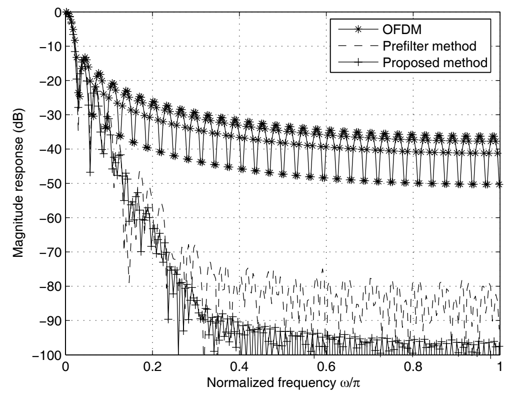

Figure 4: Magnitude response of the receive prototype filters.

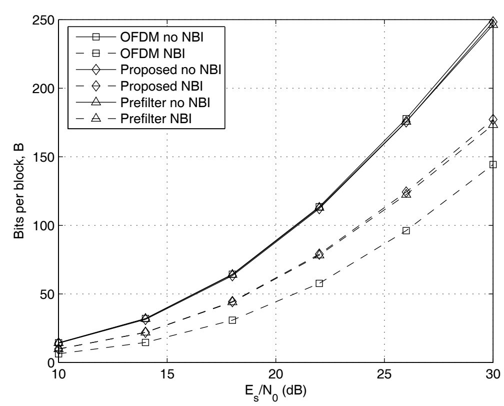

Figure 5: Comparison of bit rate.

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