Statistical Bit Allocation and Statistical Precoding for Correlated MIMO Channels With Decision Feedback

Yuan-Pei Lin and See-May Phoong

Abstract—**In this letter, we jointly consider statistical precoding and statistical bit allocation for correlated MIMO channels. Given the statistics of the channel, we derive statistical BER bounds for high and low SNR ranges. Based on the BER bounds, statistical precoder and statistical bit allocation are designed. The statistical precoder helps to expose the statistical difference among the subchannels, which is then exploited by bit allocation. Moreover, we will incorporate bit allocation in determining detection ordering for the decision feedback receiver and present a suboptimal ordering for minimizing the worst subchannel error rate. The suboptimal solution can be implemented efficiently by modifying existing fast algorithms developed for uniform bit allocation. Although the statistical precoder and bit allocation are designed for high and low SNR ranges, very good performance can be achieved for all SNR. Simulations show that the performance of the proposed system is comparable to one that has instantaneous feedback rate of around 4 bits per channel.**

I. Introduction

THE VBLAST system that employs successive interference cancellation at the receiver [1] has been shown to be very useful for MIMO transmission. The VBLAST system uses uniform bit and power allocation, and no feedback is needed. It has been extended by incorporating power allocation, bit allocation or precoding when instantaneous feedback of the current channel information is available, e.g., feedback of bit allocation [2], [3], feedback of power and bit allocation [4], and feedback of bit allocation and precoder [5]. When the channel statistics are available to the transmitter but not the current state of the channel, statistical precoder or statistical bit allocation has been designed. For example, precoders for minimizing error probability are derived in [6], [7]. The optimal precoder that minimizes the sum of mean squared error is given in [8]. In these works, a uniform bit allocation is assumed. Optimization of precoders for a fixed bit allocation has been developed in [9]. Statistical bit allocation is designed in [10] for uncorrelated channels based on capacity modeling. Statistical bit allocation for correlated channels is derived in [11] by combining antenna selection and bit loading.

In this letter, we jointly consider statistical designs of precoding and bit allocation for correlated MIMO channels. Assuming the transmitter knows the statistics of the channel, which requires only infrequent feedback, we derive the precoding matrix and bit allocation to minimize BER bounds for high and low SNR ranges. With precoding, the channel is decorrelated and the disparity among the subchannels is made more pronounced. The resulting statistical difference of subchannels is then exploited using bit allocation.[1] Moreover we will consider a near optimal detection ordering that can be implemented using existing fast algorithms. Simulations show that the performance of the suboptimal ordering is indistinguishable from that of optimal ordering. With the proposed combination of statistical precoding and bit allocation, very good performance can be obtained compared to that with statistical precoding or statistical bit allocation alone.

The sections are organized as follows. In Section II, we give the MIMO system model of the proposed system. In Section III., statistical precoder and bit allocation are derived. An efficient suboptimal detection ordering is presented. Simulation results are given in Section IV and a conclusion is given in Section V.

Notation. Boldfaced lower case letters represent vectors and boldfaced upper case letters are reserved for matrices. The notation $\mathbf{A}^\dagger$ denotes transpose-conjugate of $\mathbf{A}$. The function $E[y]$ denotes the expected value of a random variable y.

II. System Model

Consider the MIMO system with M_t transmit antennas and M_r receive antennas in Fig. 1. The channel is modeled by an $M_r \times M_t$ memoryless matrix $\mathbf{H}$ with $M_r \times 1$ additive white Gaussian channel noise $\mathbf{q}$ that has zero mean and variance N_0. The channel considered in this letter is of the form $\mathbf{H} = \mathbf{H}_w \mathbf{R}_t^{1/2}$, where $\mathbf{H}_w$ is an $M_r \times M_t$ matrix whose elements are independent Gaussian random variables with unit variance and $\mathbf{R}_t$, of dimensions $M_t \times M_t$, is the transmit correlation matrix. This model is useful for downlink transmission when the receiving antennas are well separated. Suppose the transmitter and receiver can process M substreams of symbols, where $M \le \min(M_t, M_r)$. The precoder $\mathbf{F}$ is an $M_t \times M$ matrix and the inputs are uncorrelated modulation symbols, $s_0, s_1, \ldots, s_{M-1}$, of zero mean and unit variance. The total

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[1]Statistical precoding has also been considered in [3] to minimize BER bounds. However the precoder in [3] is designed for the case when there is instantaneous feedback and the bit allocation is chosen optimally according to the current channel. In this letter, there is no instantaneous feedback and the precoder is designed for a fixed bit allocation.

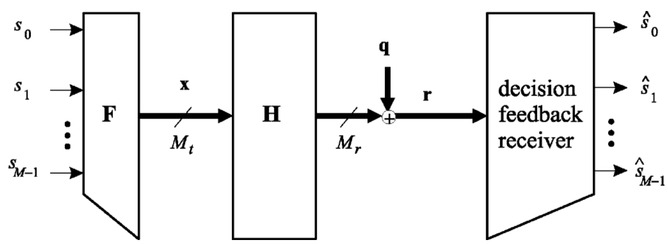

Fig. 1. MIMO system with M_t transmit antennas and M_r receive antennas.

transmission power $E[\mathbf{x}^\dagger \mathbf{x}]$ is P_t. The decision feedback receiver is zero forcing.

III. Statistical Bit Allocation and Precoder

In this section, we will consider statistical designs of bit allocation and precoder. Assuming the inputs s_k are b_k-bit QAM symbols, the number of bits transmitted per channel use R_b is thus $\sum_{k=0}^{M-1} b_k$. The kth symbol error rate (SER) is well approximated by [12]:

$$SER_k = 4(1-2^{-b_k/2})Q\left(\sqrt{\frac{3}{(2^{b_k}-1)\sigma_{e_k}^2}}\right) \quad (1)$$

where $Q(y) = 1/\sqrt{2\pi}\int_y^\infty e^{-t^2/2}dt$, $y \geq 0$, and $\sigma_{e_k}^2$ is the variance of the subchannel error $e_k = \widehat{s_k} - s_k$. When Gray code is used, the kth subchannel BER can be approximated by $BER_k \approx SER_k/b_k$. Then the average BER of the M subchannels is

$$BER \approx \frac{1}{R_b}\sum_{k=0}^{M-1} b_k BER_k \approx \frac{1}{R_b}\sum_{k=0}^{M-1} SER_k. \quad (2)$$

For the convenience of derivation, we define the function $f(y) = Q(1/\sqrt{y}), y > 0$. The function $f(y)$ is monotonically increasing and it can be verified that it is convex for $y \leq 1/3$ and concave for $y > 1/3$. Using $f(\cdot)$, we have $SER_k = 4(1-2^{-b_k/2})f\left((2^{b_k}-1)\sigma_{e_k}^2/3\right)$. In the following, we discuss the high-SNR case and the low-SNR case separately.

Consider the case the SNR is low so that all subchannels are operating in the concave region of $f(\cdot)$. In this case SER_k is a concave function of $\sigma_{e_k}^2$ for a given bit allocation. Using Jensen's inequality $E[SER_k(\sigma_{e_k}^2)] \leq SER_k(E[\sigma_{e_k}^2])$, we have

$$E[SER_k] \leq 4(1-2^{-b_k/2})f\left(\frac{1}{3}(2^{b_k}-1)\overline{\sigma}_{e_k}^2\right) \quad (3)$$

where $\overline{\sigma}_{e_k}^2 = E[\sigma_{e_k}^2]$ is the kth error variance averaged over the channel $\mathbf{H}$. Then $E[BER]$, the BER in (2) averaged over the channel, has the upper bound

$$\phi = \frac{1}{R_b}\sum_{k=0}^{M-1} 4(1-2^{-b_k/2})f\left(\frac{1}{3}(2^{b_k}-1)\overline{\sigma}_{e_k}^2\right). \quad (4)$$

Assume the transmission rate is high and b_k is large enough so that $1-2^{-b_k/2} \approx 1$ and $2^{b_k}-1 \approx 2^{b_k}$, then ϕ can be approximated by $\phi \approx (4/R_b)\sum_{k=0}^{M-1} f\left((1/3)2^{b_k}\overline{\sigma}_{e_k}^2\right)$. The concavity of $f(\cdot)$ implies

$$\phi \approx \frac{4}{R_b}\sum_{k=0}^{M-1} f\left(\frac{2^{b_k}}{3}\overline{\sigma}_{e_k}^2\right) \leq \frac{4M}{R_b} f\left(\frac{1}{3M}\sum_{k=0}^{M-1} 2^{b_k}\overline{\sigma}_{e_k}^2\right) \triangleq \phi_0. \quad (5)$$

Using $\sum_{k=0}^{M-1} b_k = R_b$, the arithmetic mean-geometric mean inequality and also the monotone increasing property of $f(\cdot)$, we can bound ϕ_0 by

$$\phi_0 \geq \frac{4M}{R_b} f\left(\frac{1}{3}2^{R_b/M}\prod_{k=0}^{M-1}\overline{\sigma}_{e_k}^{2/M}\right) \triangleq \phi_1. \quad (6)$$

The equality in (6) holds if and only if $2^{b_k}\overline{\sigma}_{e_k}^2$ are of the same value for all k. This requires

$$b_k = \frac{1}{M}\sum_{\ell=0}^{M-1}\log_2(\overline{\sigma}_{e_\ell}^2) - \log_2(\overline{\sigma}_{e_k}^2) + \frac{R_b}{M}. \quad (7)$$

In this case, the inequality in (5) also becomes an equality and thus $\phi \approx \phi_1$, a quantity that is independent of bit allocation. Note that the bound ϕ_1 depends on the product $\prod_{k=0}^{M-1}\overline{\sigma}_{e_k}^2$, which can be bounded in terms of the eigen values of $\mathbf{R}_t$ [3] when reverse detection ordering is used. Let the eigen decomposition of $\mathbf{R}_t$ be $\mathbf{U}_t\mathbf{\Lambda}_t\mathbf{U}_t^\dagger$, where $\mathbf{\Lambda}_t$ is a diagonal matrix with diagonal elements $\lambda_{t,i}$ ordered by $\lambda_{t,0} \geq \lambda_{t,1} \geq \cdots \lambda_{t,M_t-1}$. Assume $\lambda_{t,M-1} > 0$ and $M_r > M$, then $\prod_{k=0}^{M-1}\overline{\sigma}_{e_k}^2$ is bounded by

$$\prod_{k=0}^{M-1}\overline{\sigma}_{e_k}^2 \geq \prod_{k=0}^{M-1}\frac{N_0}{\frac{P_t}{M}}(M_r-k-1)^{-1}\lambda_{t,k}^{-1}. \quad (8)$$

The above inequality becomes an equality when we choose $\mathbf{F} = \sqrt{P_t/M}\mathbf{U}_{t,M}$, where $\mathbf{U}_{t,M}$ is the submatrix of $\mathbf{U}_t$ that consists of the first M columns of $\mathbf{U}_t$. Using (8), we have $\phi_1 \geq \phi_2$, where

$$\phi_2 = \frac{4M}{R_b} f\left(\frac{1}{3}2^{R_b/M}\frac{N_0}{\frac{(M_r-k-1)P_t}{M}}\prod_{k=0}^{M-1}\lambda_{t,k}^{-1/M}\right). \quad (9)$$

Therefore we have $\phi \approx \phi_2$ when $\mathbf{F}$ is chosen as

$$\mathbf{F} = \sqrt{\frac{P_t}{M}}\mathbf{U}_{t,M} \quad (10)$$

and bits allocated as in (7). The upper bound ϕ_2 is independent of the choices of bit allocation and precoder. On the other hand, when SNR is large enough so that all subchannels are operating in the convex region of $f(\cdot)$, SER_k becomes a convex function of $\sigma_{e_k}^2$. We can apply Jensen's inequality $E[SER_k(\sigma_{e_k}^2)] \geq SER_k(E[\sigma_{e_k}^2])$. Then the inequality in (3) is reversed and ϕ in (4) is a lower bound of BER. In this case we would also like to have ϕ minimized because BER cannot be small if ϕ is large. Using an approach similar to that in the low SNR case, it can be verified that ϕ is minimized if bit allocation is chosen according to (7) and precoder as in (10).

A. Statistical Bit Allocation

When $\mathbf{F}$ is chosen as in (10), the channel is decorrelated and

$$\bar{\sigma}_{e_k}^2 = \frac{N_0}{\frac{P_t}{M}}(M_r - k - 1)^{-1}\lambda_{t,k}^{-1}.$$

The statistical difference among the subchannels can then be exploited using bit allocation to have a lower error rate. The bit allocation formula in (7) in general yields real numbers. We can obtain integer bit allocation using the greedy algorithm [13]. This is because for a given precoder, ϕ is a convex function of b_k and thus the greedy algorithm can be applied. Note that such a bit allocation is obtained under the assumption that all M subchannels are used. To remove the assumption, we can compute ϕ_2 in (9) for each M_0 with $0 < M_0 \leq M$, where M_0 is the number of subchannels used, and choose the one that has the smallest ϕ_2.

B. *Detection Ordering*

The above derivation of statistical precoder and bit allocation is carried out for the case of reverse ordering. Given the precoder and bit allocation, the receiver can further optimize the detection ordering for the current channel. In the VBLAST system [1], the symbols are detected iteratively. In each iteration the symbol with the largest SNR is detected and its contribution is then removed from the received signal. Such a procedure is optimal in the sense that the worst subchannel SER is minimized. Many fast algorithms have been developed to determine the optimal ordering. The fastest known algorithm is the one given in [14]. However, when the subchannels are of different constellations, the SNR based ordering is no longer optimal; we need to take bit allocation into consideration. Let us approximate the SER in (1) as $4Q\left(\sqrt{3\rho_k}\right)$, where $\rho_k = 1/(2^{b_k} - 1)\sigma_{e_k}^2$ is the normalized SNR [15] of the kth subchannel. With this approximation, the worst subchannel SER can be minimized by maximizing $\min_k \rho_k$. It is shown in [16] that $\min_k \rho_k$ can be maximized if the ordering is such that the subchannel normalized SNR is maximized in each iteration. (Such an ordering was shown in [16] to be optimal for minimizing outage probability of capacity.) With some minor changes we can use the existing fast algorithms developed for VBLAST system to implement the decision feedback receiver. For example we can use the algorithm in [14]. In the first step of each iteration in [14], we find the subchannel with the smallest $(2^{b_k} - 1)\sigma_{e_k}^2$ instead of the subchannel with the smallest $\sigma_{e_k}^2$. The rest of the steps can be carried out as usual. The complexity is almost the same as the original algorithm. Simulations show that the performance of such a suboptimal ordering is indistinguishable from that of an optimal ordering that is obtained through an exhaustive search.

IV. Simulation Example

The exponential correlation model [17] is used to generate the random channel, in which the Hermitian matrix $\mathbf{R}_t$ is given by $[\mathbf{R}_t]_{mn} = \gamma^{n-m}$, for $0 \leq m \leq n < M_t$, where γ is the correlation coefficient between neighboring antennas. In the following Monte Carlo simulations, $M_r = 6$, $M_t = 4$, $R_b = 8$, $M = 4$, and 10^6 channel realizations are used.

The BER of the proposed *stat*istical Bit allocation and Precoding system (statBP) is shown in Fig. 2 for correlated channels with $\gamma = 0.3, 0.5$ and 0.7, respectively. Applying the greedy algorithm for each of these three cases, we obtain the bit allocation respectively as (3, 3, 2, 0), (4, 3, 1, 0), and (5, 3, 0, 0). The bit allocation is more skewed for more a correlated channel. We have shown the BER performance of the statBP system for three different orderings: VBLAST, suboptimal and optimal orderings. With VBLAST ordering, the symbols are detected using the SNR criterion as in the VBLAST

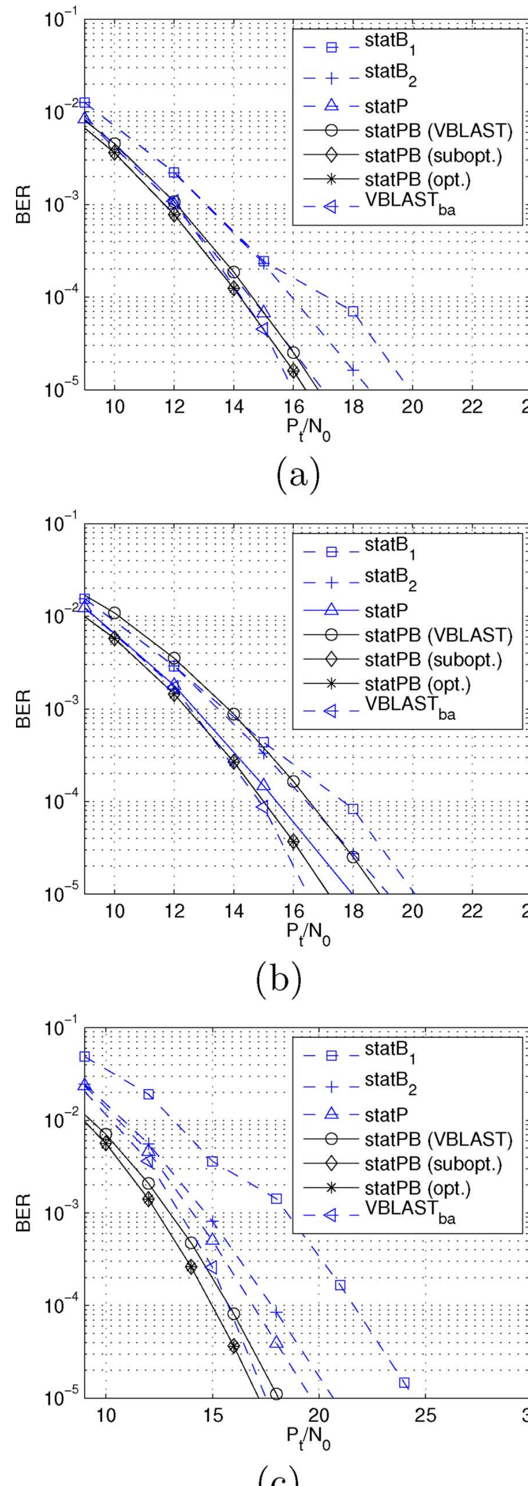

Fig. 2. BER performance for (a) $\gamma = 0.3$, (b) $\gamma = 0.5$, and (c) $\gamma = 0.7$.

system, irrespective of the constellations used. The optimal ordering is found by exhausting all possible orderings to minimize the worst symbol error rate. The curves of the proposed suboptimal ordering and optimal ordering overlap completely. The VBLAST ordering, whose complexity is almost the same as suboptimal ordering, has a performance loss ranging from 0.5 dB to 1.5 dB, depending on the bit allocation used.

Also shown in the plots are the system with statistical precoder and uniform bit allocation (statP) [8], and the two systems with statistical bit allocation [10] (statB_1) and [11] (statB_2) but no precoding. For the low-correlation case ($\gamma = 0.3$), the gain of statBP is small. However, the gap widens as correlation increases. For example, for $\gamma = 0.7$ and $\mathrm{BER} = 10^{-4}$, the gain of using statBP with suboptimal ordering is around 1.8 dB compared to the systems that use only statistical precoding or statistical bit allocation. With only precoding, the statistically bad subchannels are still loaded with bits and they dominate the overall error rate performance. With only bit allocation, the disparity among the subchannels are not exposed and we cannot take full advantage of bit allocation. For comparison, we have also shown the BER of the QR based VBLAST system with instantaneous feedback of bit allocation (VBLAST_{ba}) proposed in [5]. The bit allocation codebook is obtained by constraining b_k in a manner similar to that in [5]: b_k are even, $b_0 \geq 2$, and $b_1, b_2, b_3 \geq 0$. In this case there are 20 codewords in the codebook; $\log_2 20 \approx 4.32$ feedback bits are used for each channel realization. We can see that at $\mathrm{BER} = 10^{-4}$ the performance of the statBP system with suboptimal ordering is comparable to that of VBLAST_{ba}, which has instantaneous feedback of around 4 bits per channel.

V. Conclusion

In this letter, we proposed the use of joint statistical precoding and bit allocation for correlated MIMO channels. We have derived statistical precoder and bit allocation based on BER bounds. With the statistical precoder, the channel is decorrelated, which leads to more pronounced statistical disparity among the subchannels. The disparity is then exploited using bit allocation. We have also proposed a suboptimal constellation-dependent ordering that can be implemented using existing fast algorithms. Simulations showed that the performance of the proposed statistical design and efficient suboptimal ordering is comparable to one that has access to instantaneous feedback of around 4 bits per channel.

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